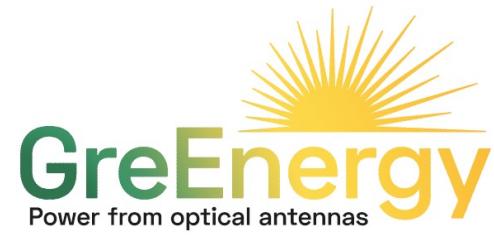


Ancona, Italy 31 May & 01 June 2022



Multiphysics Simulation of Nanodevices

Luca Pierantoni, Davide Mencarelli

Università Politecnica delle Marche, Ancona, Italy

l.pierantoni@staff.univpm.it, d.mencarelli@staff.univpm.it



UNIVERSITÀ
POLITECNICA
DELLE MARCHE

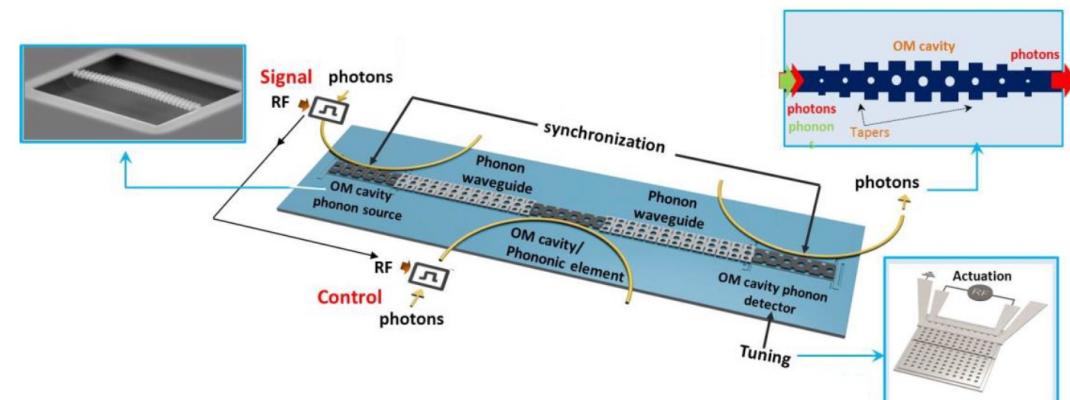
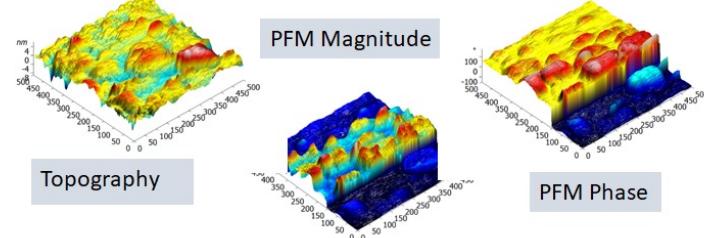
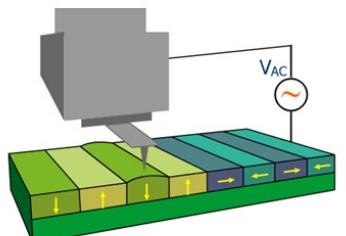
OUTLINE

- Research Areas and Activities**
- Nano- /smart material in devices/systems**
- The computational platform for the multi-physics modeling**
- Examples**

Electromagnetics – Nanotechnology group : Research Areas and Activities

- Analysis EM + Quantum / thermal transport in nano-structures
- Computational platform for the multi-physics modeling of nano-to-meso-scale systems
- Atomistic (*ab-Initio*) simulations
- Molecular Dynamics (MD) simulations

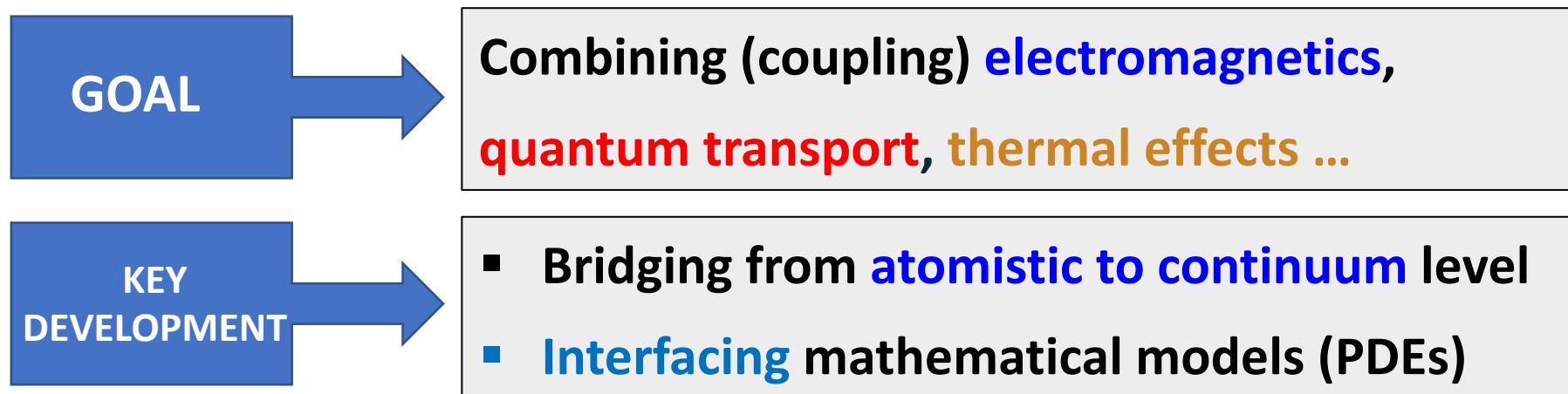
- Modeling/design of opto-mechanical systems
- Opto-electronics and microscopy



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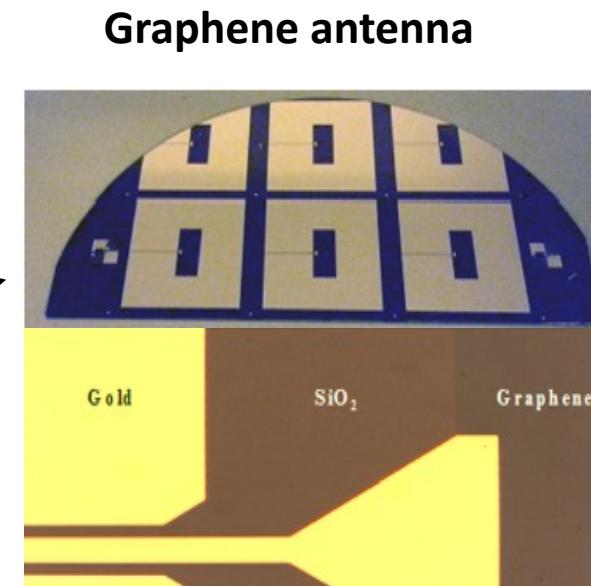
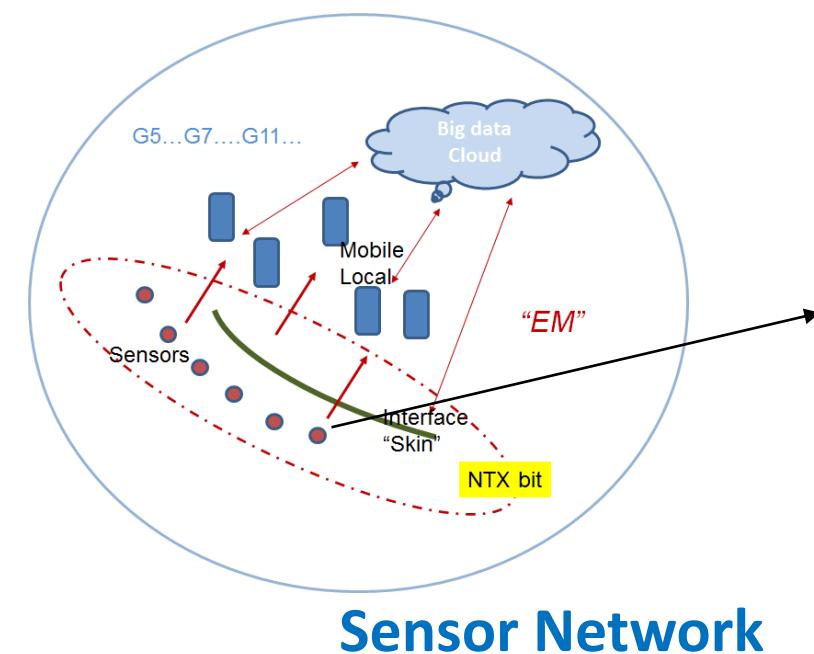
Electromagnetics – Nanotechnology group : Research Areas and Activities

- Analysis **EM + Quantum / thermal transport** in nano-structures
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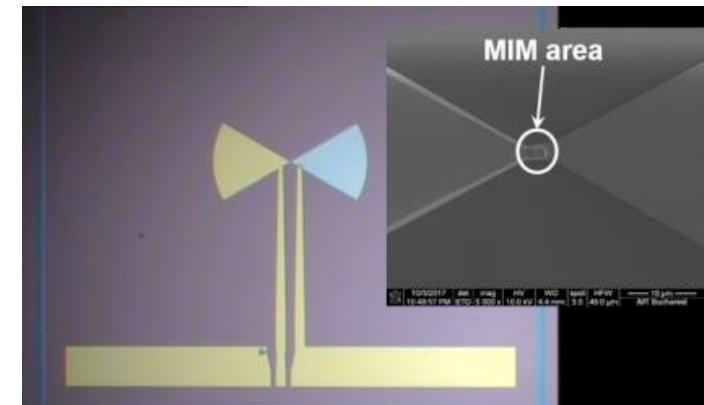
... a typical scenario where the mutiphysics modeling becomes necessary a technological platform incorporating nano- / smart-materials

- nano-structured material regions (CNT, graphene etc...)
- embedded in micro-/mm regions
- Extreme multi-scale: geometrical/electrical aspect ratios
- Multi-physics (EM+transport+thermal+...) phenomena



Graphene thickness:
< 1 nm

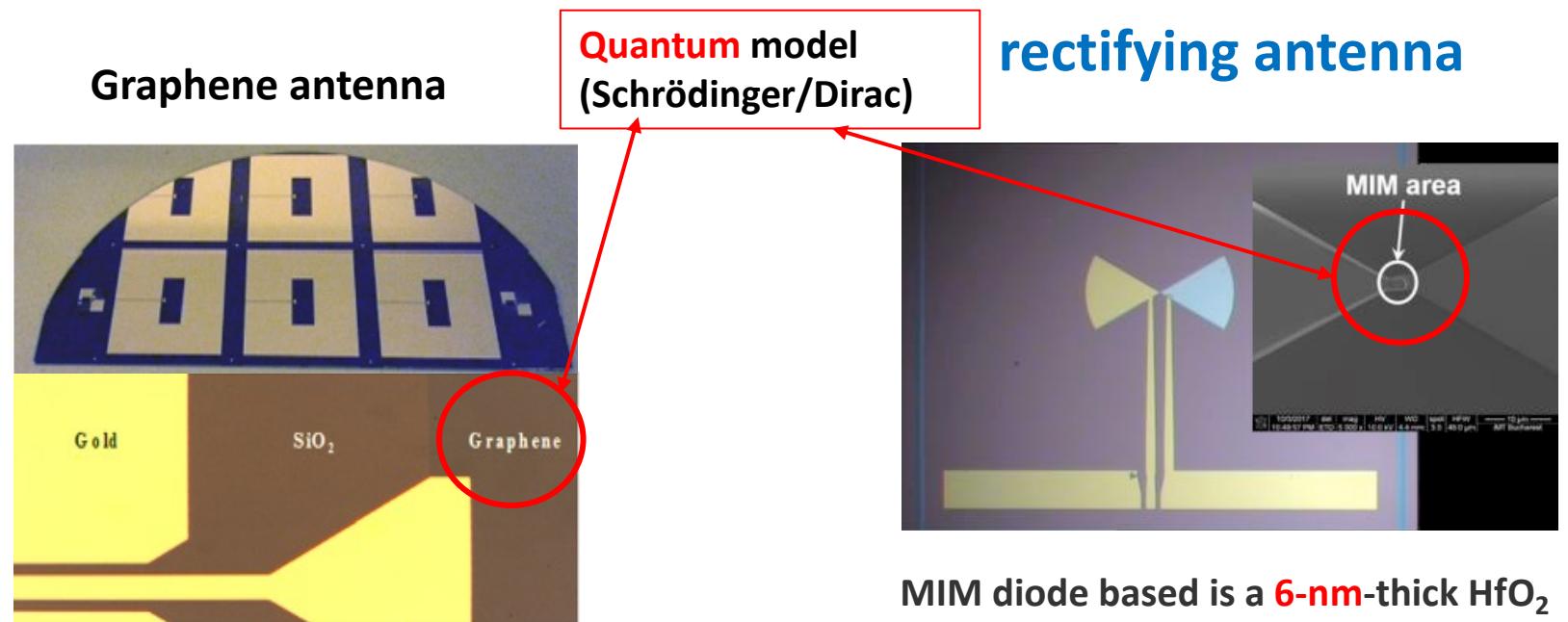
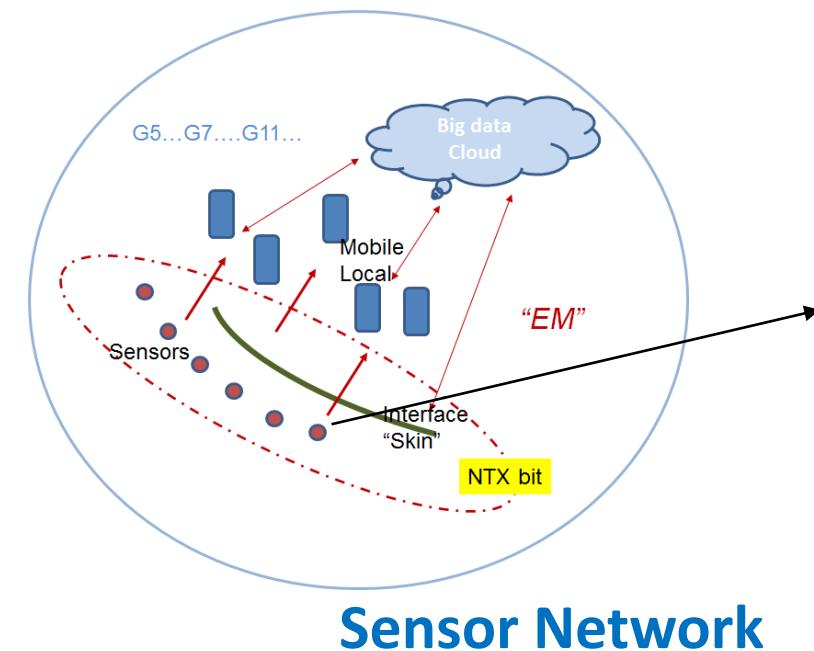
rectifying antenna



MIM diode based is a **6-nm**-thick HfO_2

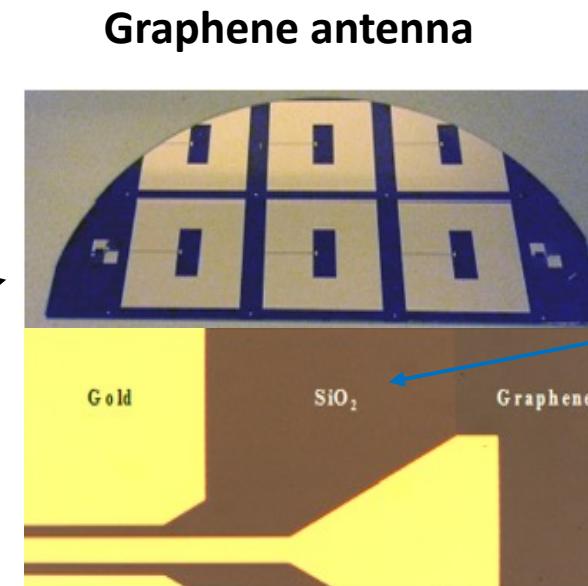
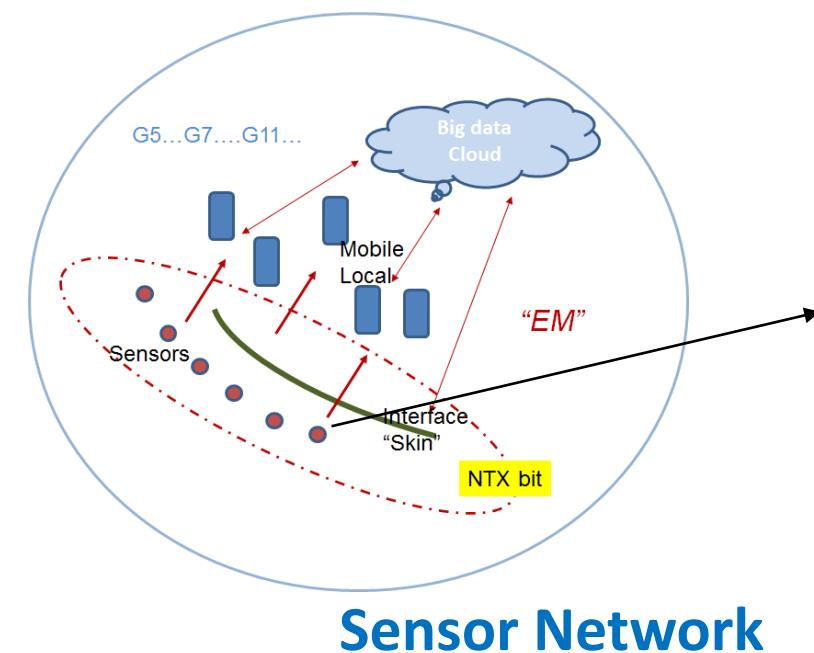
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- embedded in **micro-/mm** regions (**EM** models)
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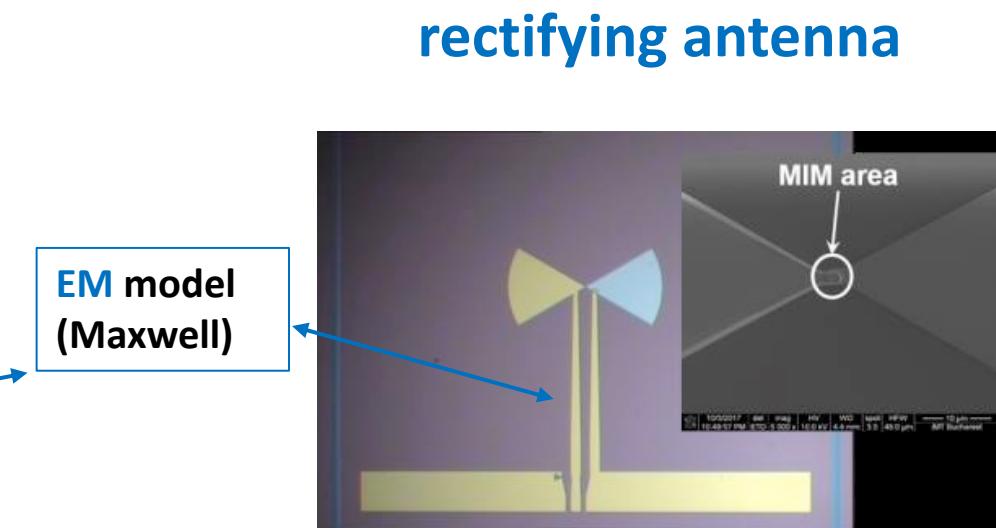


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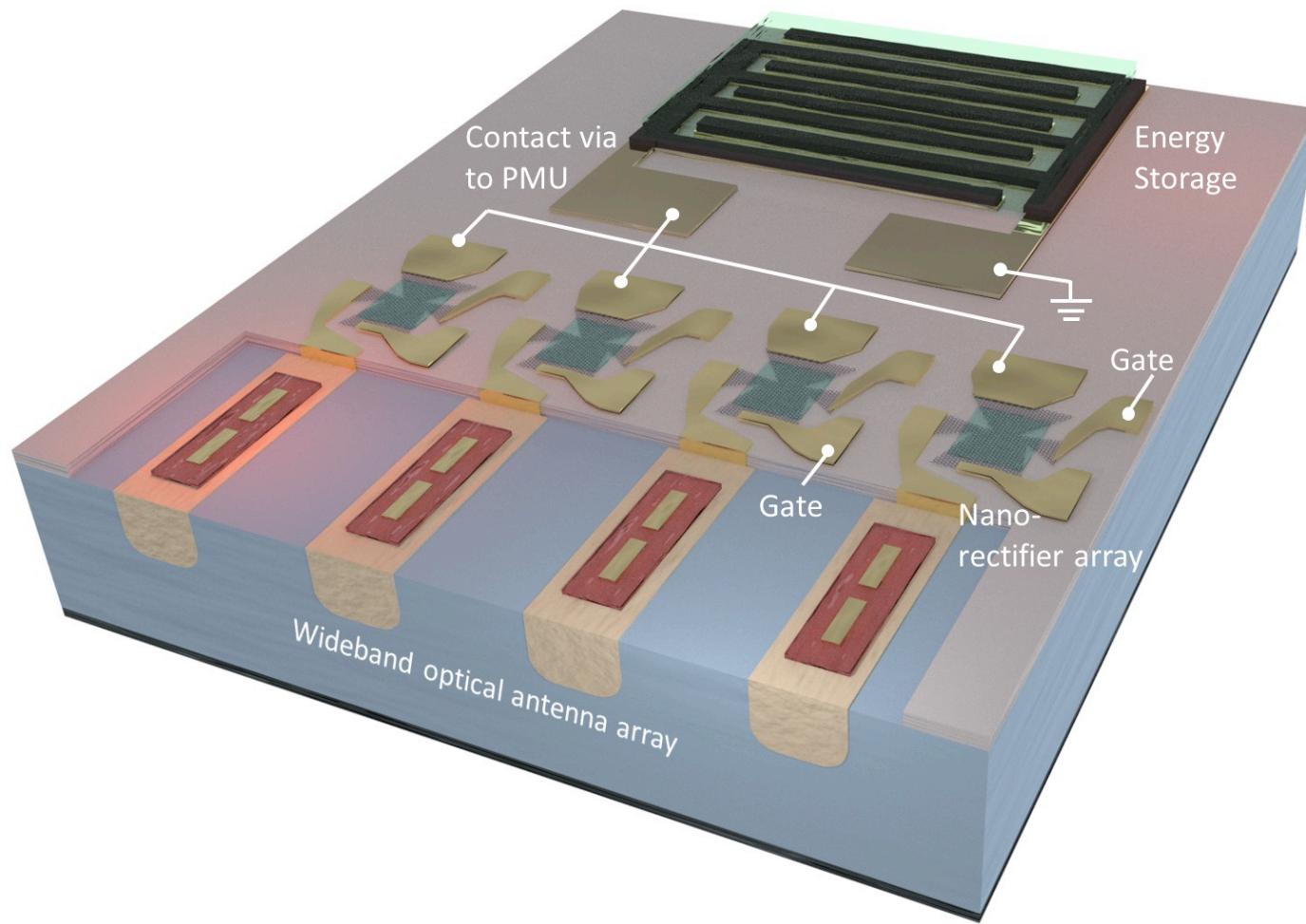
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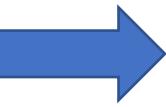
M. Aldrigo, M. Dragoman, M. Modreanu, et al., 2018,
<https://doi.org/10.1109/TED.2018.2835138>

GreEnergy scenario: rectenna + diode + supercap + ...



Schematic view of the future combined system architecture

**KEY
DEVELOPMENT**

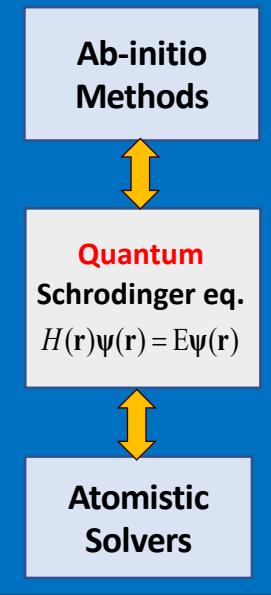
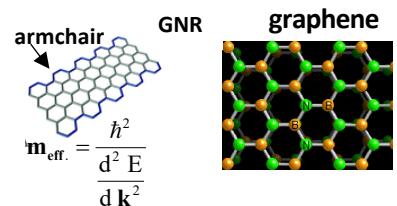
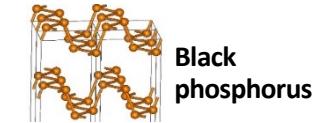
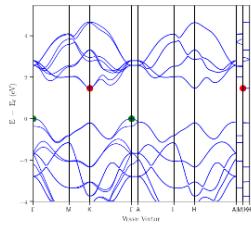


- Bridging from atomistic to continuum level
- Interfacing mathematical models (PDEs)

KEY DEVELOPMENT

- Bridging from **atomistic** to **continuum** level
- Interfacing mathematical models (PDEs)

ATOMISTIC LEVEL



THE BRIDGE

$$\begin{aligned}\varepsilon(\omega, \mathbf{k}) \\ \mu(\omega, \mathbf{k}) \\ \sigma(\omega, \mathbf{k}) \\ m_{\text{effective}}\end{aligned}$$

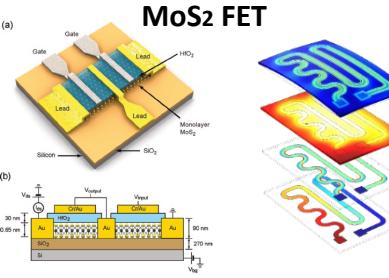
$$\mathbf{J}(\mathbf{r}) = \int d^2 r' \sigma_{HO}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')$$

$\sigma_{HO} = \sigma_{\text{Hafnium Oxide}}$
constitutive
eqs./relations

FULL-WAVE SOLVERS

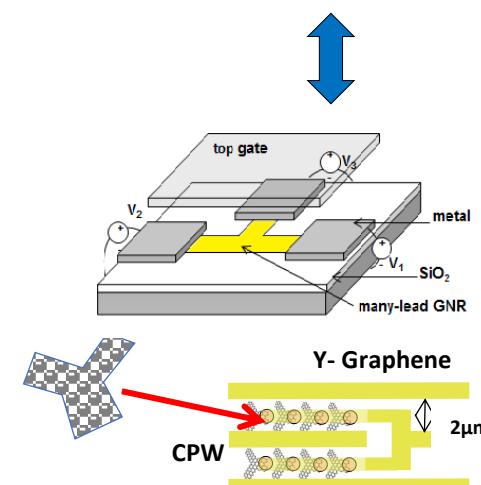
$$\begin{aligned}\nabla \times \mathbf{E}(\mathbf{r}, t) &= -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t) \\ \nabla \times \mathbf{H}(\mathbf{r}, t) &= \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}(\mathbf{r}, t)\end{aligned}$$

Electromagnetic: Maxwell



Devices Systems Interfaces

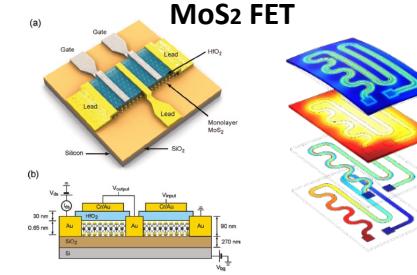
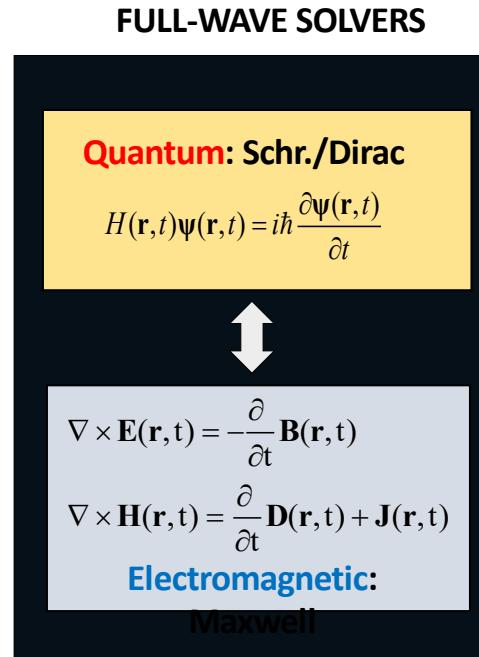
CONTINUUM LEVEL



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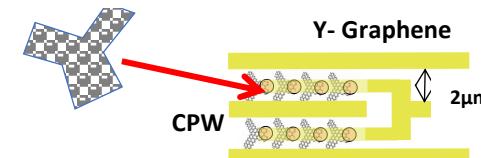
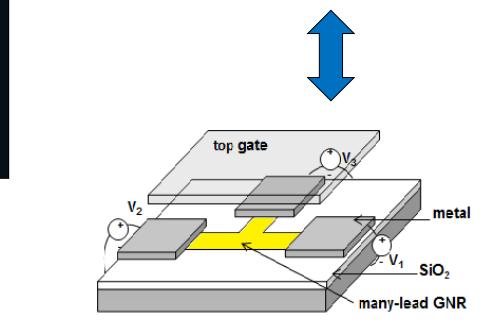
KEY DEVELOPMENT

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**Devices
Systems
Interfaces**

CONTINUUM LEVEL

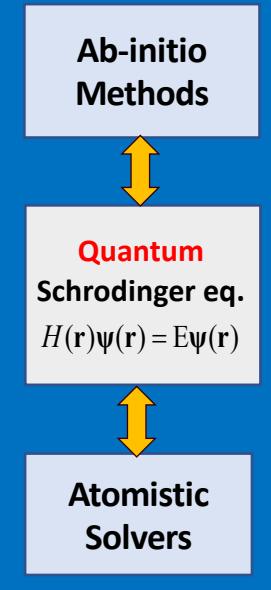
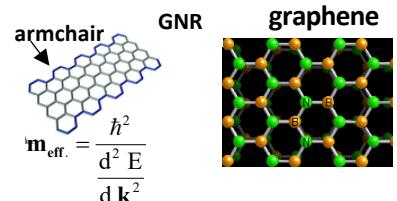
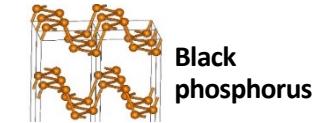
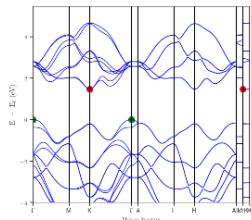


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KEY DEVELOPMENT

- Bridging from **atomistic** to **continuum** level
- Interfacing **mathematical models** (PDEs)

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$$\begin{aligned}\varepsilon(\omega, \mathbf{k}) \\ \mu(\omega, \mathbf{k}) \\ \sigma(\omega, \mathbf{k}) \\ m_{\text{effective}}\end{aligned}$$

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constitutive
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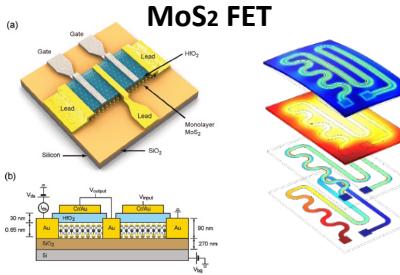
Quantum: Schr./Dirac

$$H(\mathbf{r}, t)\psi(\mathbf{r}, t) = i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t)$$

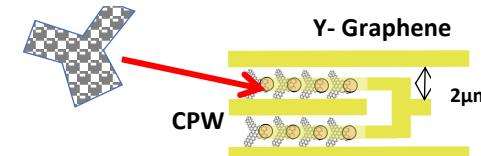
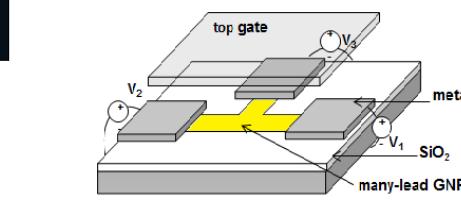
$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}(\mathbf{r}, t)$$

Electromagnetic:
Maxwell



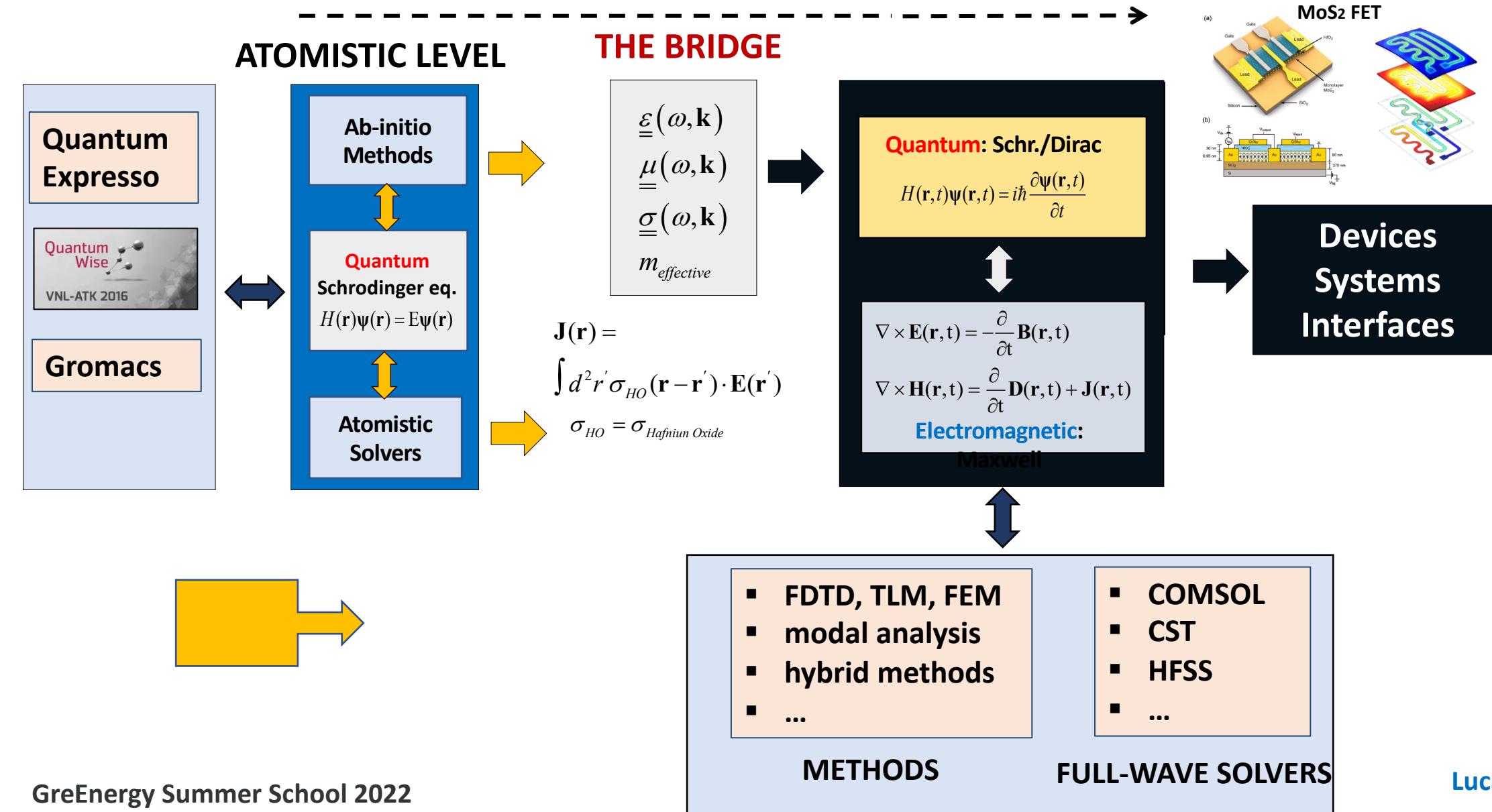
Devices
Systems
Interfaces

CONTINUUM LEVEL



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The theoretical-computational platform

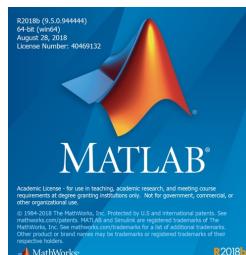


Computational platform: new home-made interface COMSOL - MATLAB

- Quantum transport, Schrödinger/Dirac
- SM, NEGF
- DFT analysis

- DC – AC analysis
- Full-wave EM analysis of FET in linear regime
- Equivalent circuits

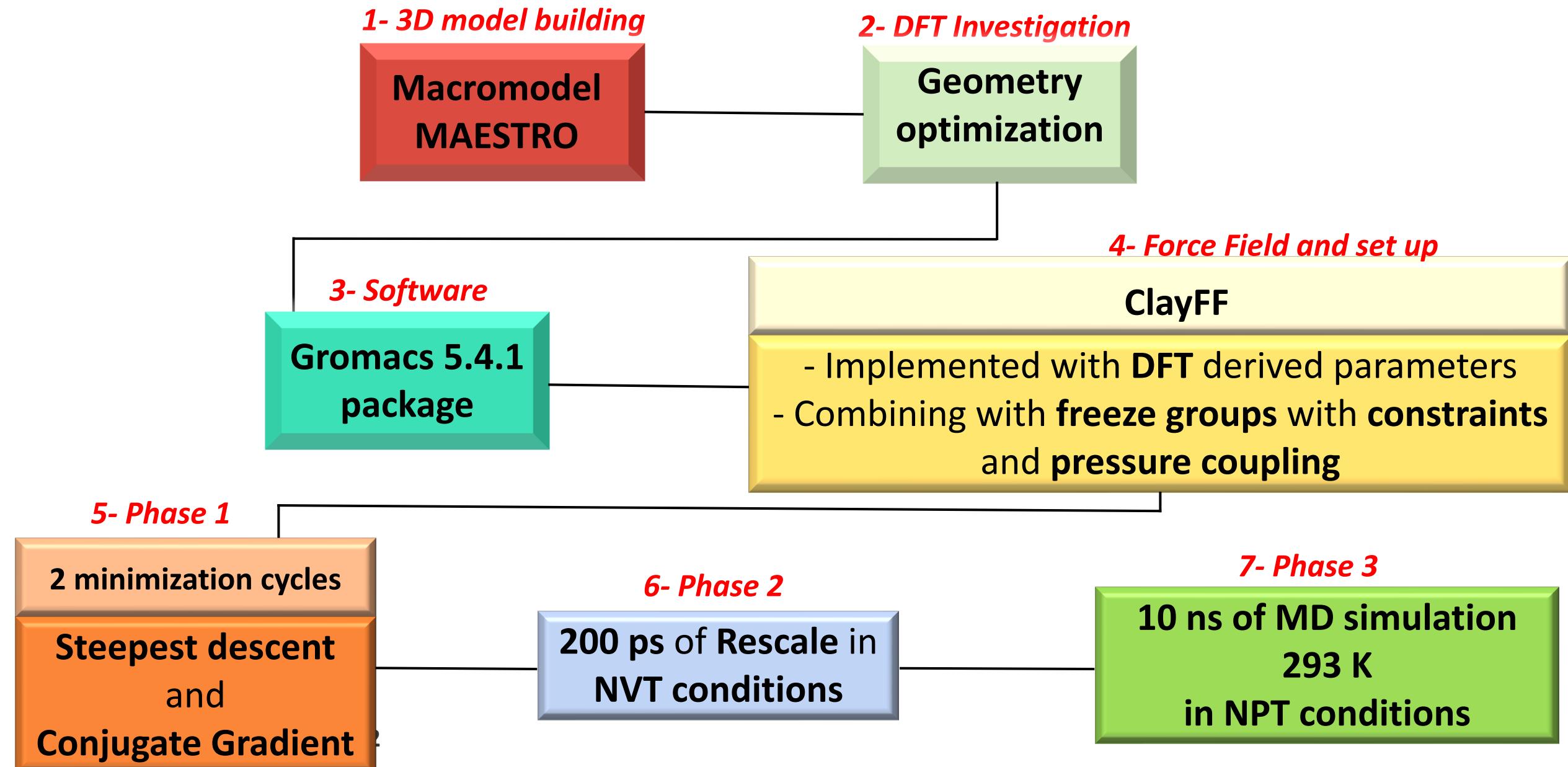
- In-house software:
Fortran, Matlab, C, etc.
- Quantum W, Gromacs



- COMSOL multiphys
- CST Microwave S.
- HFSS Ansoft
- EM3DS Univpm

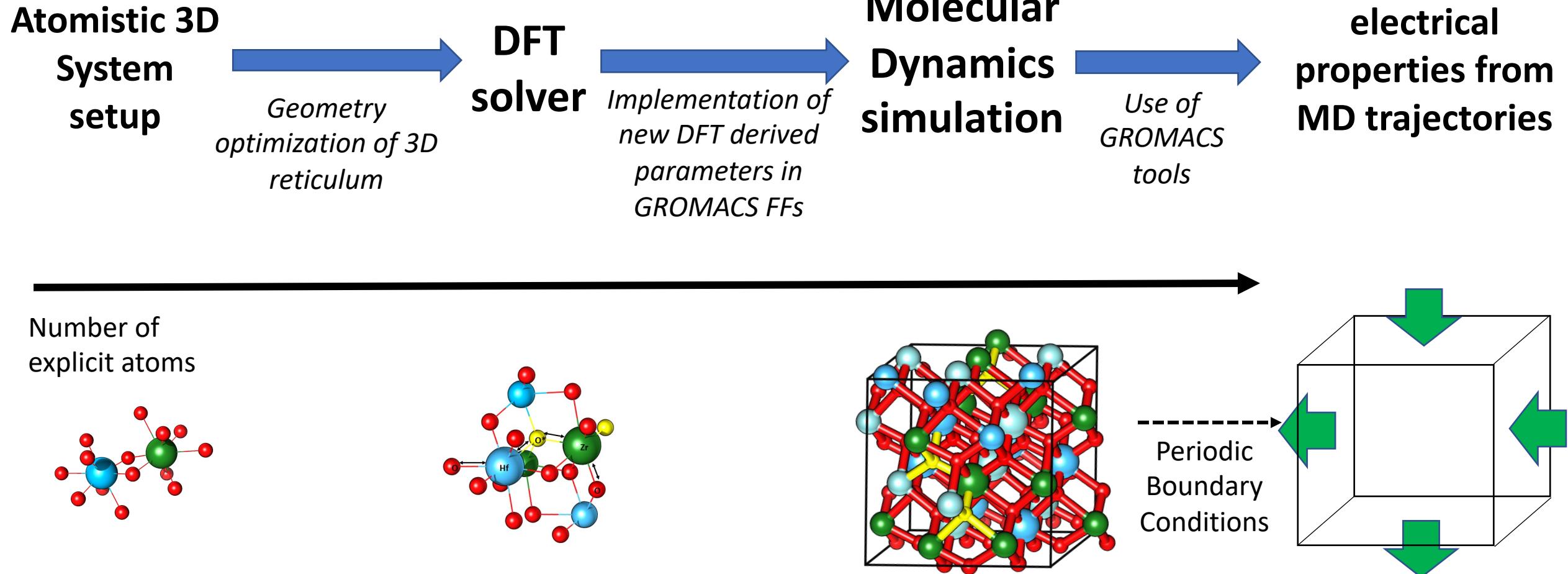


Block scheme of the Atomistic Simulations Method



DFT and Molecular Dynamics simulations

In silico methods based on combined DFT-MD simulation



Quantum and EM Models at continuum level (ii)

- 1) Quantum models are coupled to Maxwell eqs. in time- and/or freq. domain
- 2) combined electromagnetic-transport phenomena
- 3) Multi-scale numerical techniques (FEM, FDTD, ...)

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t)$$
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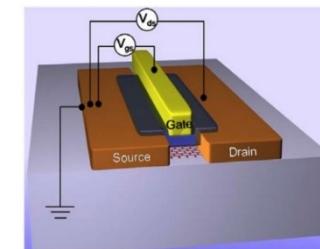
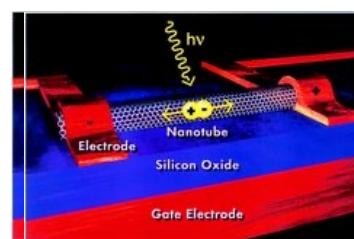
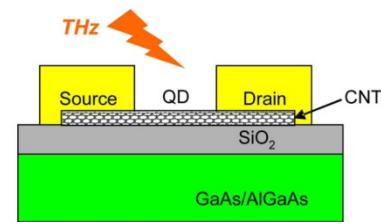
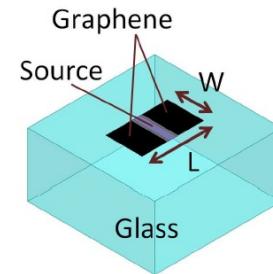
Electromagnetic: Maxwell



$$H(\mathbf{r}, t)\psi(\mathbf{r}, t) = i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t}$$

Quantum: Schrödinger/Dirac

- interaction particles-EM field - transient
- Quantum dots, quantum wells
- Ballistic electronics
- non linear devices
- Spintronics
- photodetectors
- ...



Examples

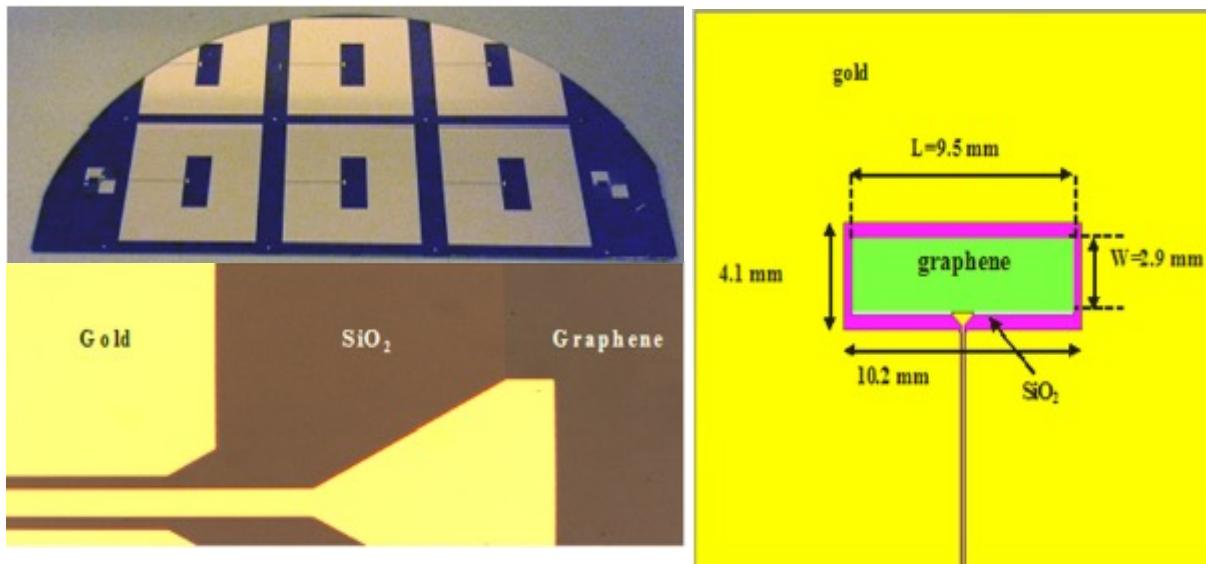
- **Graphene Antenna**
- **MoS₂ FET**
- **Schrödinger – Poisson eqs.: CNT FET**
- **Dirac – Maxwell eqs.: Ballistic Ratchet effect on graphene**

Examples

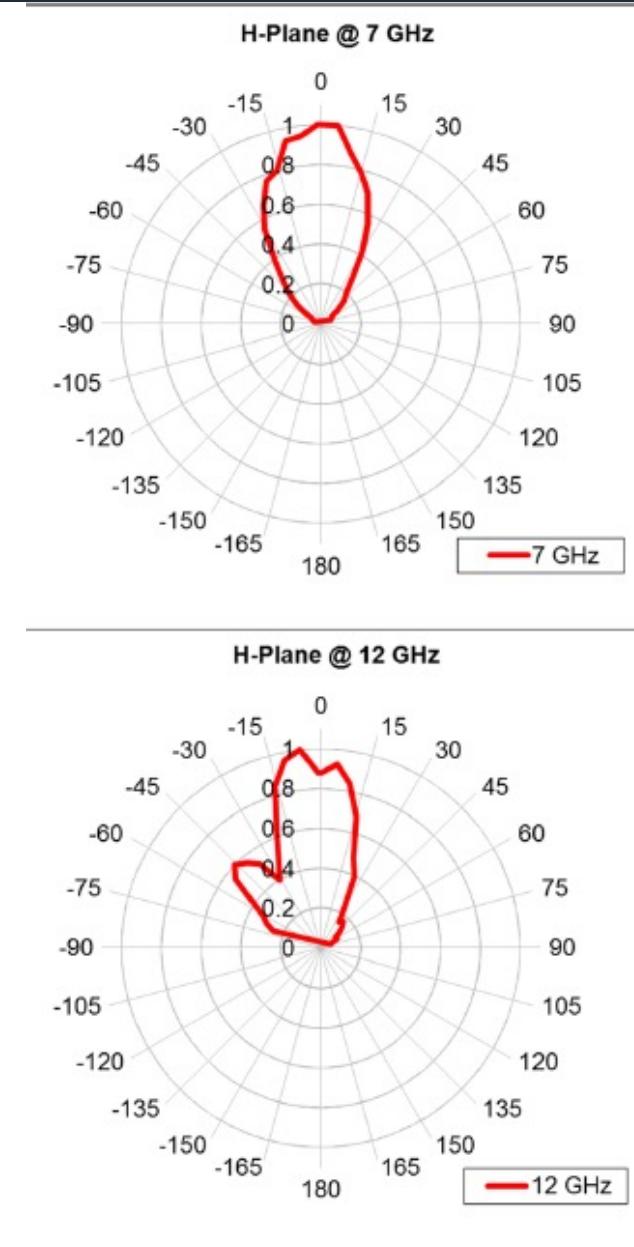
- **Graphene Antenna**
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Tunable Graphene-based Antenna

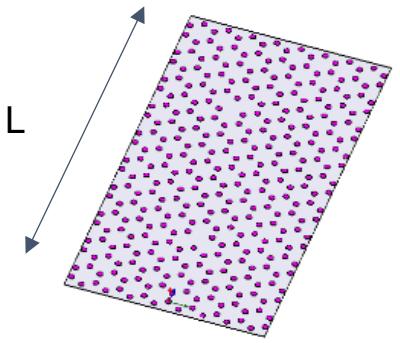
M. Dragoman, L. Pierantoni, et al.,
APPLIED PHYSICS LETTERS 106, 2015



- microwave slot antenna in a CPW based on graphene
- Antennas fabricated on a high-resistivity Si wafer
- 300 nm SiO₂ layer
- A CVD grown graphene layer is transferred on the SiO₂
- Reflection parameter can be tuned by a DC voltage
- 2D radiation patterns in the X band (8–12 GHz)



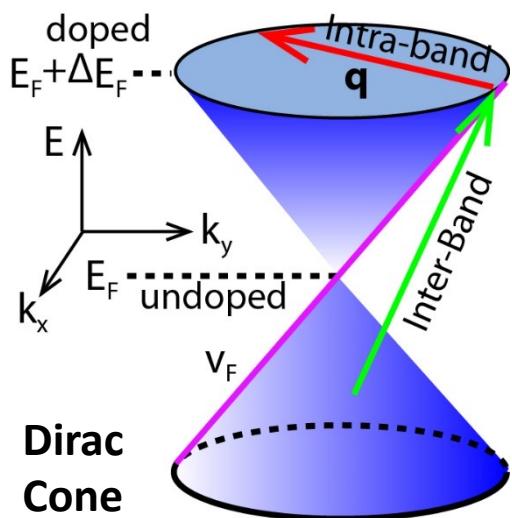
Graphene: the constitutive relation is based on the Kubo-Drude model (strictly valid for monolayer/on air)



Graphene can be described by a surface conductivity tensor to be inserted in a EM computation

$$\underline{\underline{\sigma}}_{graphene} = \underline{\underline{\sigma}}' + j\underline{\underline{\sigma}}''$$

$$\underline{J} = \underline{\underline{\sigma}}_G \underline{\underline{E}}$$



Kubo-Drude Formulation (monolayer/on air)

$$\begin{aligned} \sigma_{\pm} &= \frac{i e^2}{\pi \hbar} \frac{1}{\hbar \omega \pm i\gamma} \int_0^{\infty} d\varepsilon \varepsilon \left(\frac{\partial f_D(\varepsilon)}{\partial \varepsilon} - \frac{\partial f_D(-\varepsilon)}{\partial \varepsilon} \right) && \text{intraband} \\ &- \frac{i e^2}{\pi \hbar} \frac{1}{\hbar \omega \pm i\gamma} \int_0^{\infty} d\varepsilon \left[\frac{f_D(\varepsilon) - f_D(-\varepsilon)}{1 - \left(\frac{2\varepsilon}{\hbar \omega \pm i\gamma} \right)^2} \right] && \text{interband} \end{aligned}$$

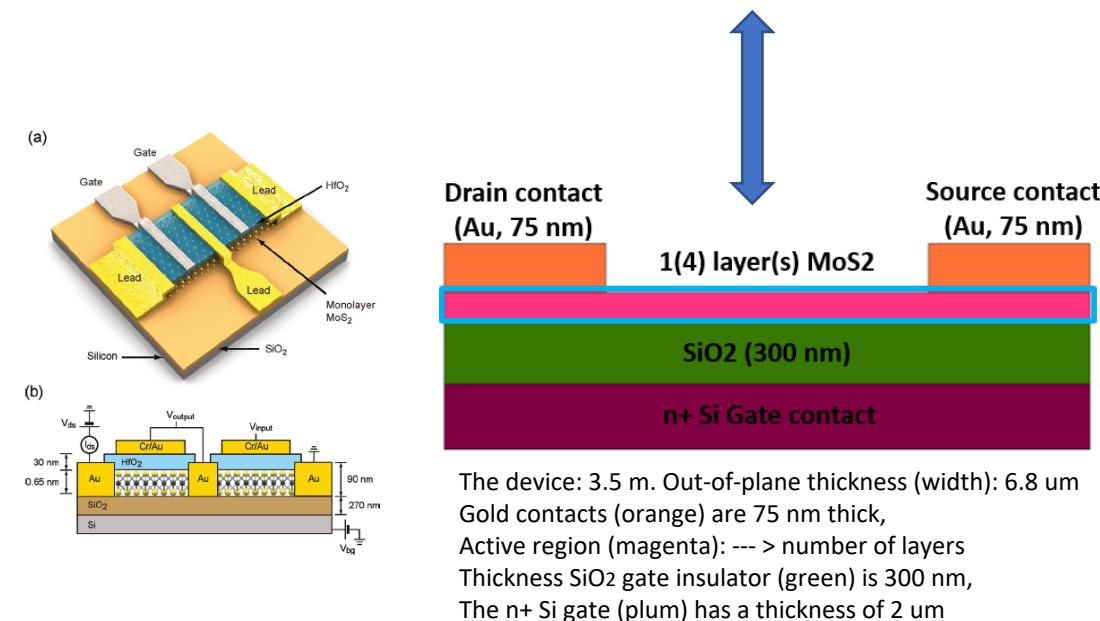
$$f_D(\varepsilon) = \frac{1}{1 + e^{\frac{\varepsilon - \mu}{kT}}} \quad \text{Fermi - Dirac Distribution} \quad \gamma = 2\pi/\tau$$

Examples

- Graphene Antenna
- **MoS₂ FET**
- Schrödinger – Poisson eqs.: CNT FET
- Dirac – Maxwell eqs.: Ballistic Ratchet effect on graphene

MoS₂ – based Field Effect Transistor (FET)

- GOAL: derivation of the MoS₂ channel permittivity (conductivity)



MoS₂

- Very HIGH ON/OFF ratios $>10^7$ (vs 10^6 of CMOS)
- Lower mobility (many defects) vs. graphene/
- Eg = 1.8 eV

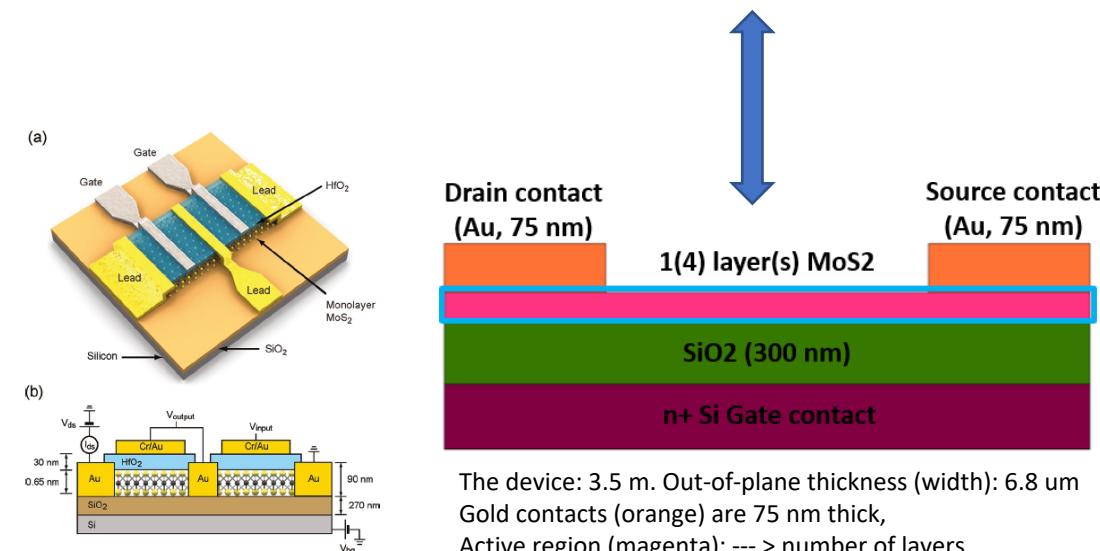
Theoretical-computational route

1. study of the material (MoS₂) at the atomistic level
2. derivation of constitutive relations (CR)
3. insertion of the CR in the full-wave solver (COMSOL)
4. Coupling of Poisson and transport (drift) eqs. using the semiconductor physics module by COMSOL

- Hafnium-Zirconium Oxide ($HfxZr(1-x)O_2$, x = 0.3)
- as a substrate ferroelectric material
- high tunability
- CMOS technology compatibility

MoS₂ – based Field Effect Transistor (FET)

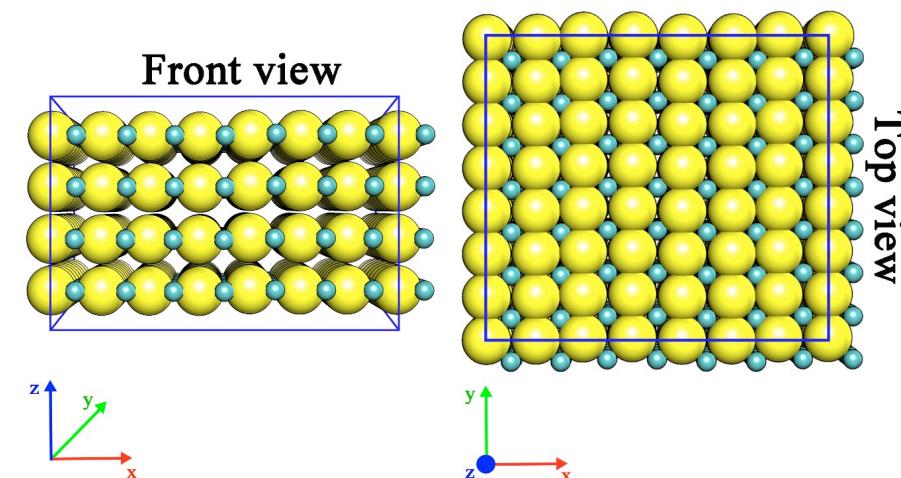
■ GOAL: derivation of the MoS₂ channel permittivity (conductivity)



MoS₂

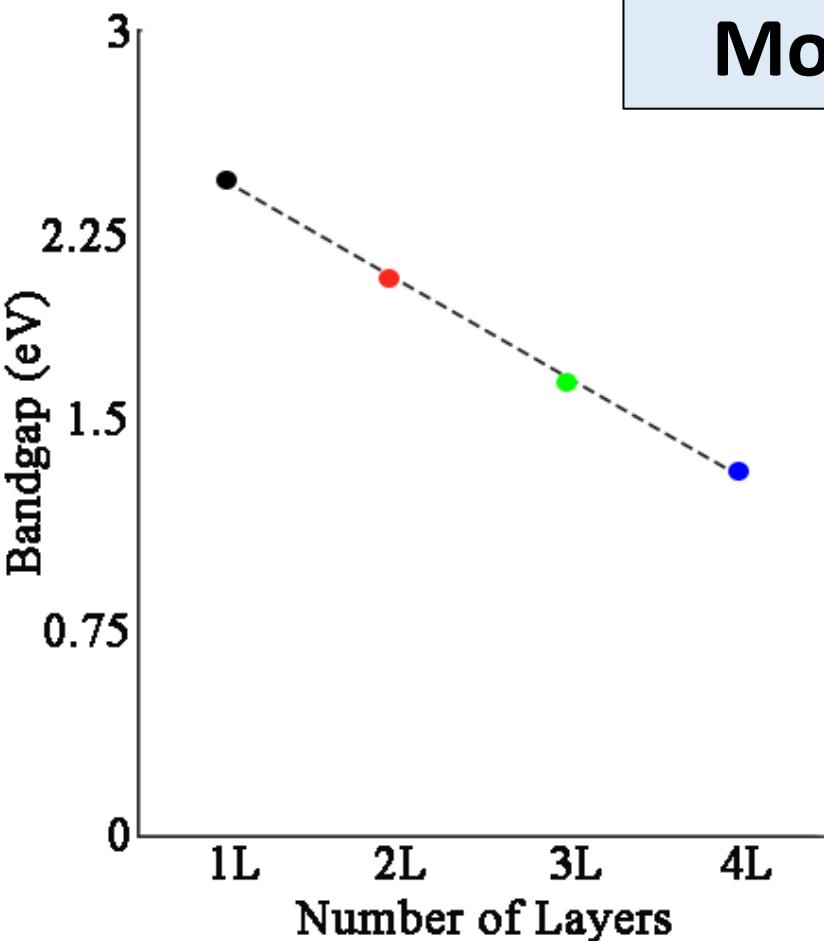
- Very HIGH ON/OFF ratios $>10^7$ (vs 10^6 of CMOS)
- Lower mobility (many defects) vs. graphene/
- Eg = 1.8 eV
- Electron affinity: 4.7 eV
- Electron (hole) mobility: $10 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ ($10 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$)
- Electron (hole) effective mass: $0.5 \times m_e$ ($0.5 \times m_e$)
- Defect n-type doping: $1.5 \times 10^{18} \text{ cm}^{-3}$

Atomistic Simulations

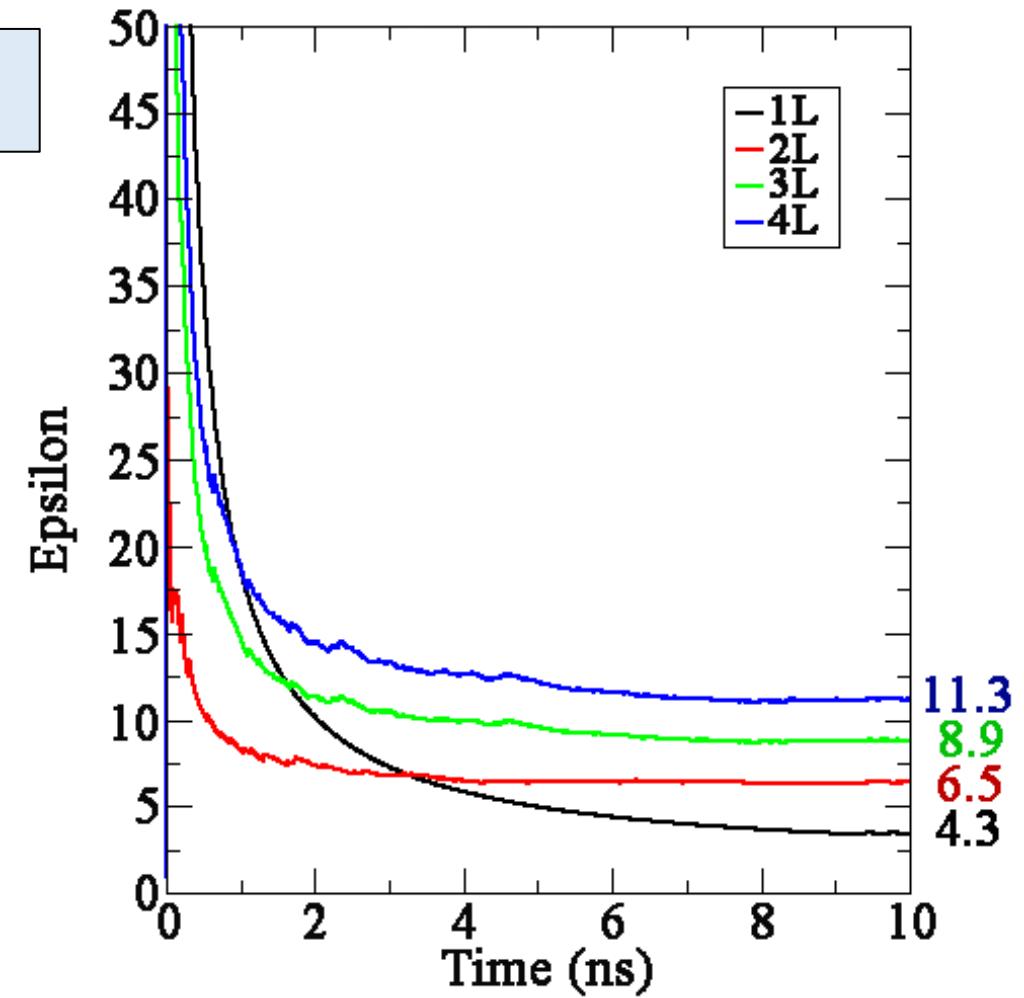


- front view / top view of 4L MoS₂
- Mo atoms in green sticks
- S atoms in yellow

possibility of simulating defects and particular contacts with the substrate

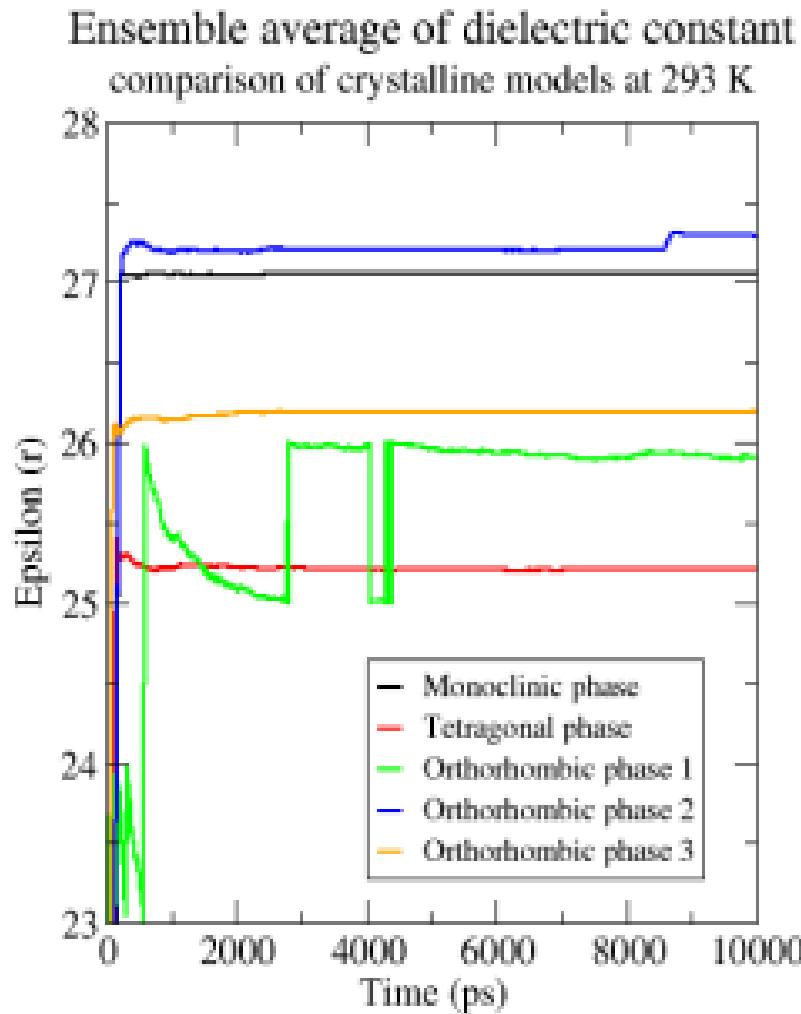


Bandgaps decreased with increasing MoS₂ layers' number [Zhao, C M Wei, L Yang, M Y Chou. Phys Rev Lett. 2004;92(23):236805]



A direct correlation between the number of layers and the dielectric constant value was observed

HfO₂ : Comparison of DFT simulations vs. data reported in the literature



27.242 → *Orthorhombic phase 2* (bandgap= 3.393 eV; from lit.)

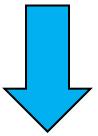
27.083 → *Monoclinic phase* (bandgap= 4.017 eV; from lit.)

26.236 → *Orthorhombic phase 3* (bandgap= 4.227 eV; from lit.)

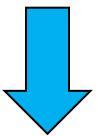
25.907 → *Orthorhombic phase 1* (bandgap= 4.413 eV; from lit.)

25.244 → *Tetragonal phase* (bandgap= 4.668 eV; from lit.)

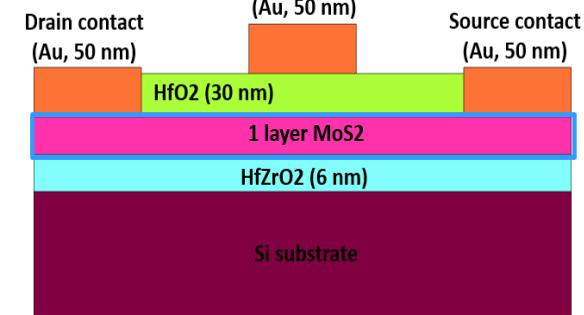
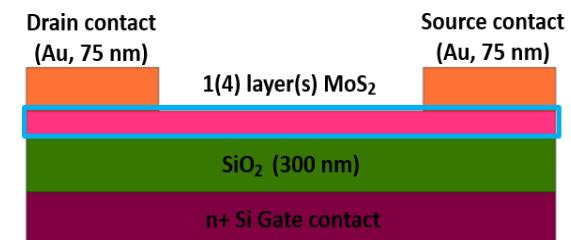
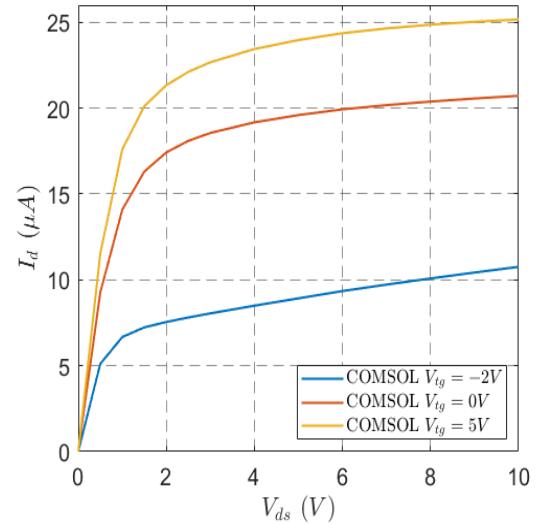
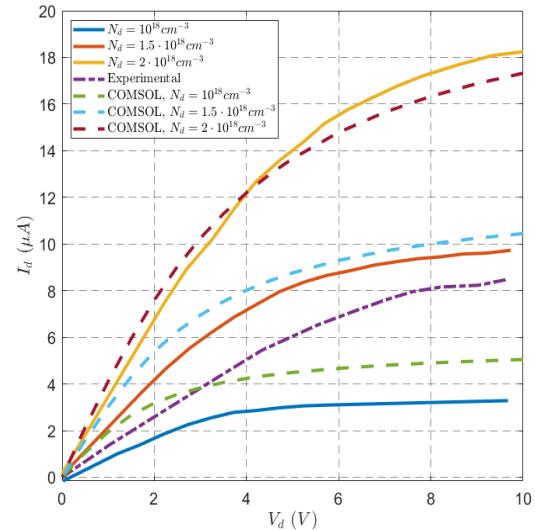
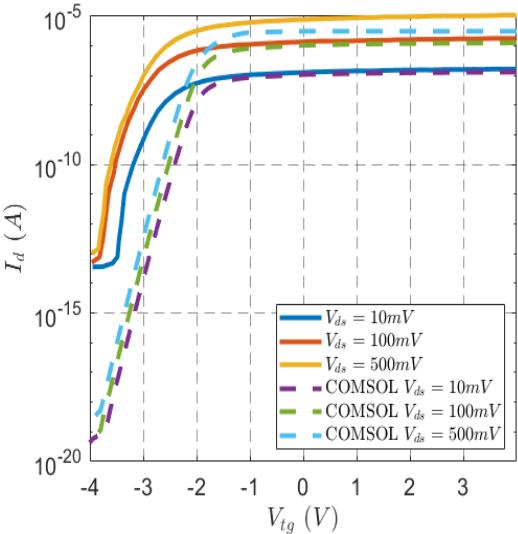
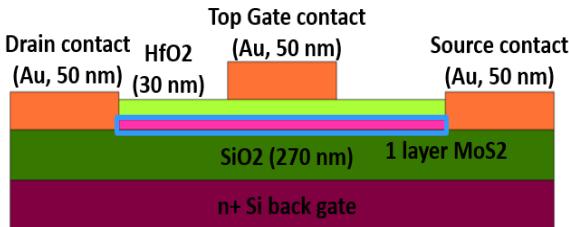
Atomistic Simulations



Derivation of constitutive relations



FEM Full-Wave Modeling



Examples

- Graphene Antenna
- MoS₂ FET
- **Schrödinger – Poisson eqs.: CNT FET**
- Dirac – Maxwell eqs.: Ballistic Ratchet effect on graphene

Schrödinger – Poisson eqs. coupling

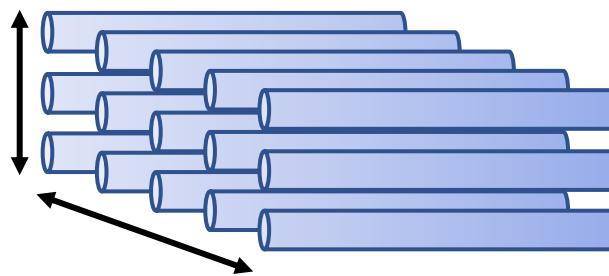
Modeling of sensors, FETs, quantum dots, ...



Quantum capacitance, effective
transconductance, etc....

Schrödinger – Poisson eqs. coupling

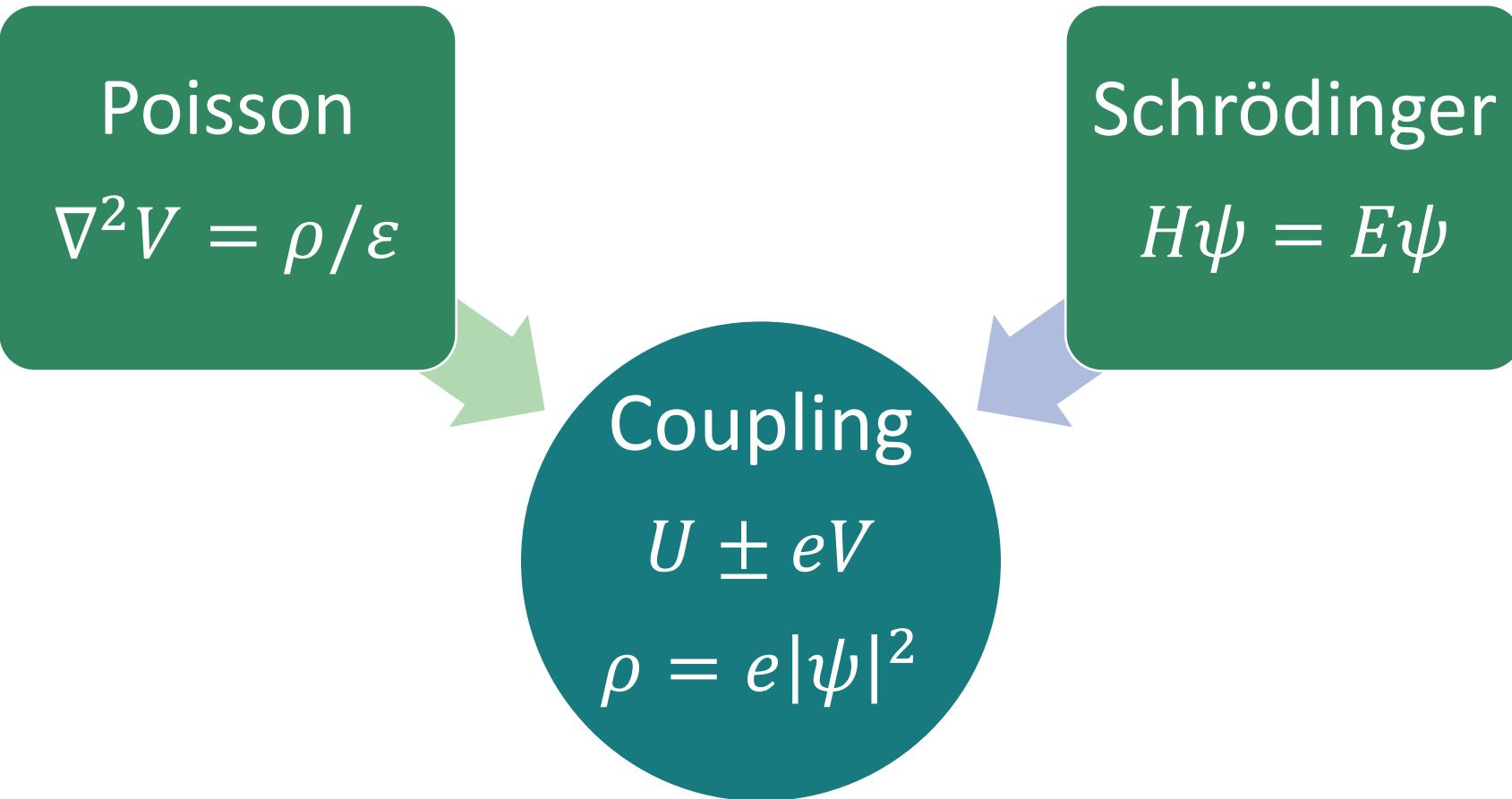
- Possibility to evaluate **multiwall/multichannel structures**
- **Rigorous analysis (no equivalent circuit approximations)**



CNT Matrix

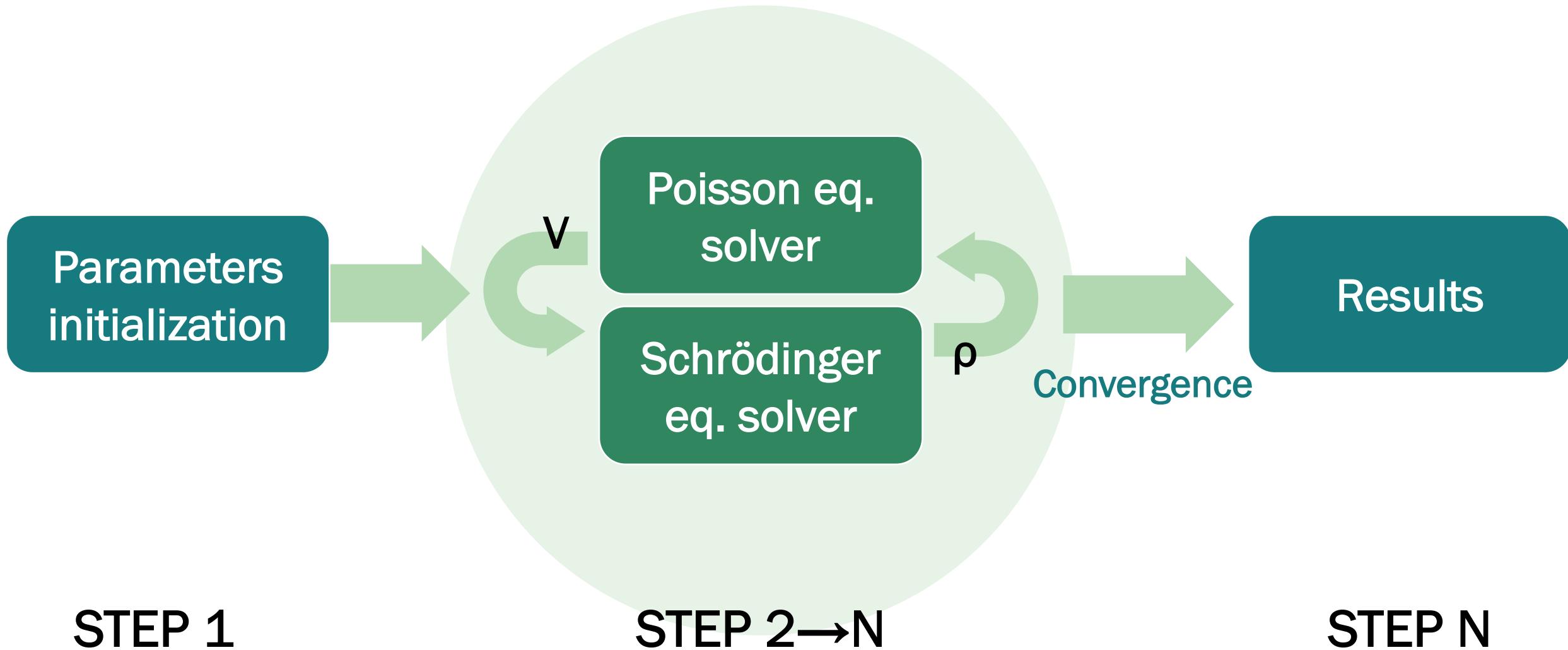
Simulation method

3D FEM
full-wave
solver



Transmission
line matrix
cascade

Simulation workflow



Schrödinger Equation model for CNT

describes the quantum mechanical behaviour of a charge carrier

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \psi(\mathbf{r}, t) = \left(\frac{1}{2m} \hat{\mathbf{p}}^2 + V(\mathbf{r}) \right) \psi(\mathbf{r}, t)$$

wave function $\psi(\mathbf{r}, t)$

$\hbar = \frac{h}{2\pi}$

$h=6.26*10^{-34} \text{ [J*sec]}$
Plank's constant

$V(r)$ is the potential barrier

$\int_{-\infty}^{+\infty} |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = 1$

m is the “effective mass” of the particle

$\hat{\mathbf{p}} = -i\hbar \nabla$ is the kinematic momentum

CNT-based Devices: self-consistent solution of the combined Poisson- Schrödinger eqs.

Schrödinger equation

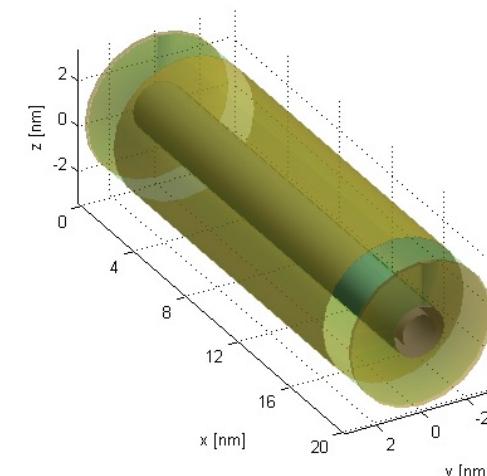
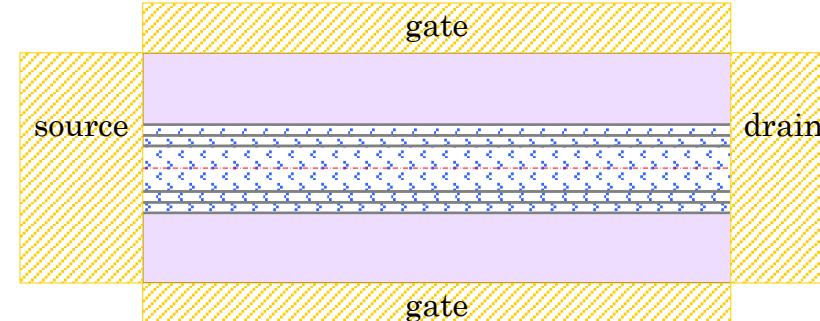
$$\nabla^2 \psi = -\frac{2m_0}{\hbar^2} [E + V] \psi$$

$$\nabla^2 V = \frac{\rho_T}{\epsilon}$$

normalization condition

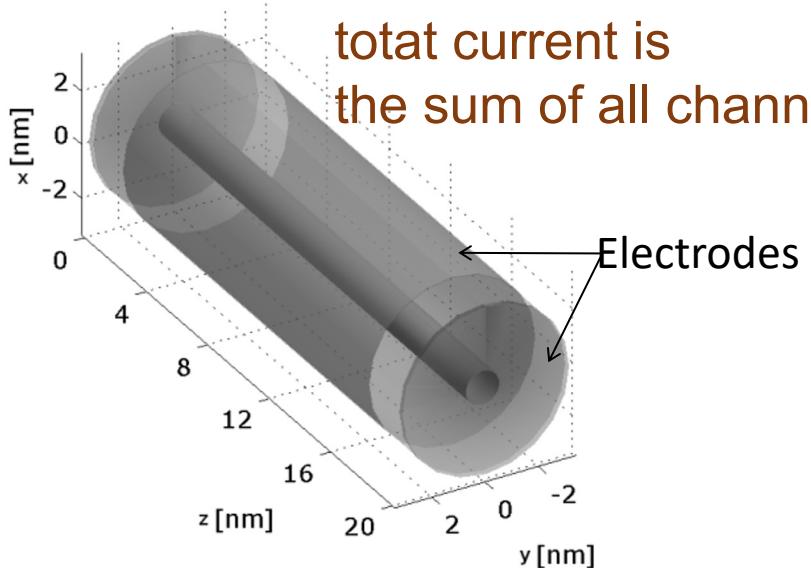
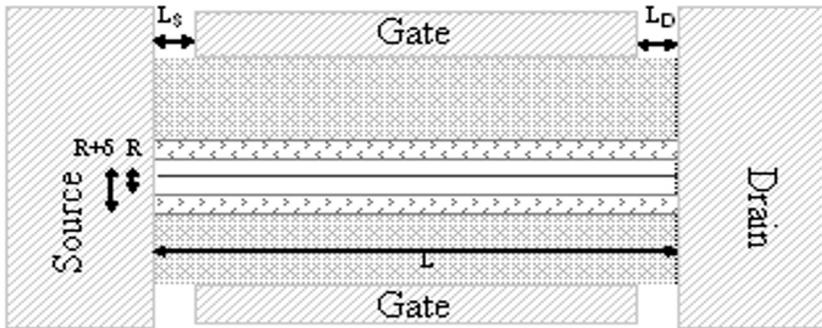
$$q \int |\psi(\mathbf{r}, E)|^2 dE = \rho_T$$

Poisson equation



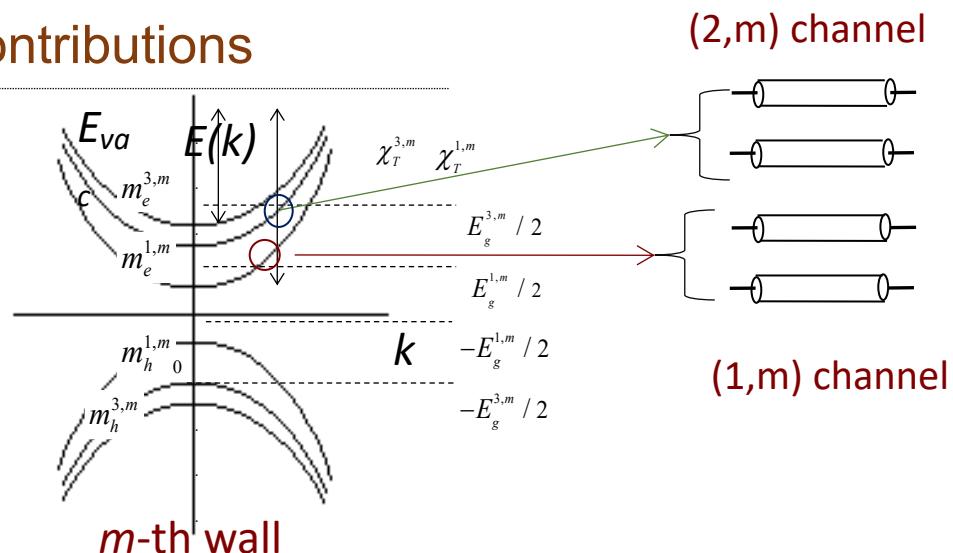
- Schrödinger equation for all the transport channels (modes)
- multi-wall and multi-band coherent carrier transport
- V is the electrostatic potential satisfying the Poisson equation

The current is computed by the Landauer-Büttiker formula



$$I_{h,e}^{n,m} = \frac{4e}{h} \int (f_{h,e}^S - f_{h,e}^D) T_{h,e}^{n,m} dE$$

$$I = \sum_{n,m} (I_h^{n,m} + I_e^{n,m})$$



Analysis of CNT Transistors

multi-wall and multi-band coherent carrier transport

$$\frac{d^2\Psi_{h,e}^{n,m}}{dz^2} = -\frac{2m_{h,e}^{n,m}}{\hbar^2} \left(E - U_{h,e}^{n,m}(z) \right) \Psi_{h,e}^{n,m}$$

m-th wall n-th sub-band

Schrödinger equations

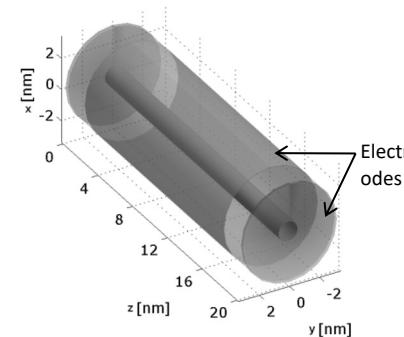
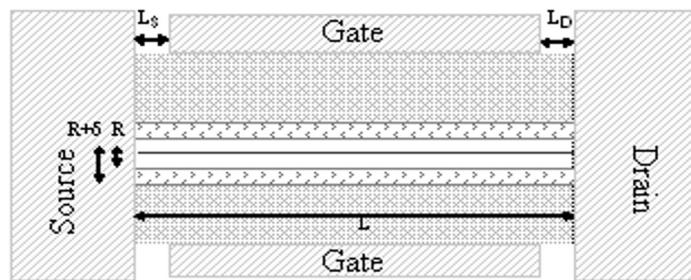
Transmission Line approach



$$\frac{d^2V}{d\rho^2} + \frac{1}{\rho} \frac{dV}{d\rho} + \frac{dV}{dz^2} = -\frac{Q}{\epsilon}$$

Poisson equation

FEM approach



The Schrödinger equation is written for each individual transport channel

V is the electrostatic potential satisfying the Poisson equation

iterative solution

$$\frac{d^2\Psi_{h,e}^{n,m}}{dz^2} = -\frac{2m_{h,e}^{n,m}}{\hbar^2} \left(E - U_{h,e}^{n,m}(z) \right) \Psi_{h,e}^{n,m}$$

potential energy of the (n-m) channel

$$\frac{d^2V}{d\rho^2} + \frac{1}{\rho} \frac{dV}{d\rho} + \frac{dV}{dz^2} = -\frac{Q}{\epsilon}$$

$$Q(V) = Q \equiv \Psi_{h,e}^{n,m}$$

$$V(Q) = V$$

$$Q = \frac{e}{2\pi} \frac{\delta(\rho - R)}{\rho} q \quad q = \int |\Psi_e^D|^2 + |\Psi_e^S|^2 dE - \int |\Psi_h^D|^2 + |\Psi_h^S|^2 dE$$

$$\begin{cases} U_e^{n,m}(z) = \tilde{U}_e^{n,m}(z) - eV(R + \delta_m, z) \\ U_h^{n,m}(z) = \tilde{U}_h^{n,m}(z) + eV(R + \delta_m, z) \end{cases} \quad \begin{cases} \tilde{U}_e^{n,m}(z) = E_{vac} - e\chi_T^{n,m} \\ \tilde{U}_h^{n,m}(z) = \tilde{U}_e^{n,m}(z) - E_g^{n,m} \end{cases} \quad \begin{cases} V(R_G, z) = V_g - \Phi_g \\ V(R + \delta_m, 0) = V_s - \Phi_s \\ V(R + \delta_m, L) = V_d - \Phi_d \end{cases}$$

E_{vac} is the vacuum energy

R_G radius of the gate electrode
 Φ_g, Φ_d, Φ_s , the work functions

$E_g^{n,m}$ *n*-th energy gap of the *m*-th wall
 $\chi_T^{n,m}$ is the electron affinity

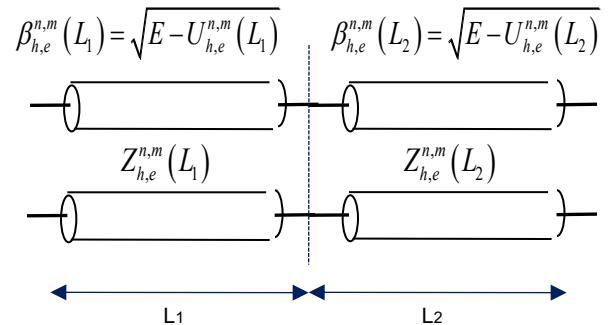
Transmission Line approach

telegrapher's equations

$$\begin{cases} \Psi_{h,e}^{n,m} = V_{h,e}^{n,m+} e^{-j\beta_{h,e}^{n,m} z} + V_{h,e}^{n,m-} e^{j\beta_{h,e}^{n,m} z} \\ \frac{\partial \Psi_{h,e}^{n,m}}{\partial z} = I_{h,e}^{n,m+} e^{-j\beta_{h,e}^{n,m} z} + I_{h,e}^{n,m-} e^{j\beta_{h,e}^{n,m} z} \end{cases}$$

$$\beta_{h,e}^{n,m} = \sqrt{E - U_{h,e}^{n,m}}$$

propagation constants



$$= \frac{V_{h,e}^{n,m}}{jZ_{h,e}^{n,m}}^+ e^{-j\beta_{h,e}^{n,m} z} - \frac{V_{h,e}^{n,m}}{jZ_{h,e}^{n,m}}^- e^{j\beta_{h,e}^{n,m} z}$$

$$Z_{h,e}^{n,m} = 1 / \beta_{h,e}^{n,m}$$

characteristic impedances

$$\Psi_{h,e}^{n,m} = \begin{cases} A_{h,e}^{n,m} e^{-j\beta_{h,e}^{n,m,S} z} + B_{h,e}^{n,m} e^{j\beta_{h,e}^{n,m,S} z} & z < 0 \text{ (source)} \\ C_{h,e}^{n,m} e^{-j\beta_{h,e}^{n,m,D} z} + D_{h,e}^{n,m} e^{j\beta_{h,e}^{n,m,D} z} & z > L \text{ (drain)} \end{cases}$$

solutions

Normalization condition

$$(A_{h,e}^{n,m})^2 = \frac{2m_{h,e}^{n,m}}{\pi\hbar^2} \frac{f_{h,e}^{n,m,S}}{\beta_{h,e}^{n,m,S}}$$

$$D_{h,e}^{n,m2} = \frac{2m_{h,e}^{n,m}}{\pi\hbar^2} \frac{f_{h,e}^{n,m,D}}{\beta_{h,e}^{n,m,D}}$$

Analysis of CNT Transistors

multi-wall and multi-band coherent carrier transport

$$\frac{d^2\Psi_{h,e}^{n,m}}{dz^2} = -\frac{2m_{h,e}^{n,m}}{\hbar^2} \left(E - U_{h,e}^{n,m}(z) \right) \Psi_{h,e}^{n,m}$$

m-th wall n-th sub-band

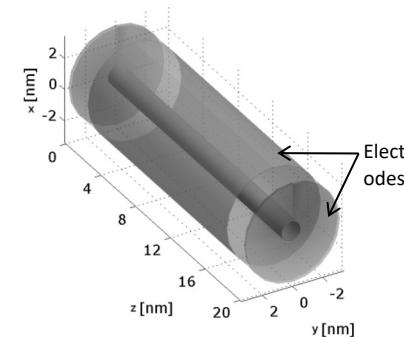
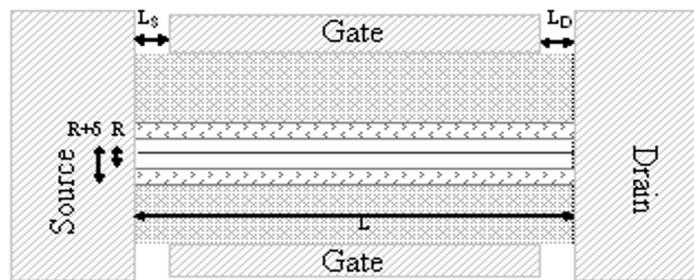
Schrödinger equations

Transmission Line approach

$$\frac{d^2V}{d\rho^2} + \frac{1}{\rho} \frac{dV}{d\rho} + \frac{dV}{dz^2} = -\frac{Q}{\epsilon}$$

Poisson equation

FEM approach



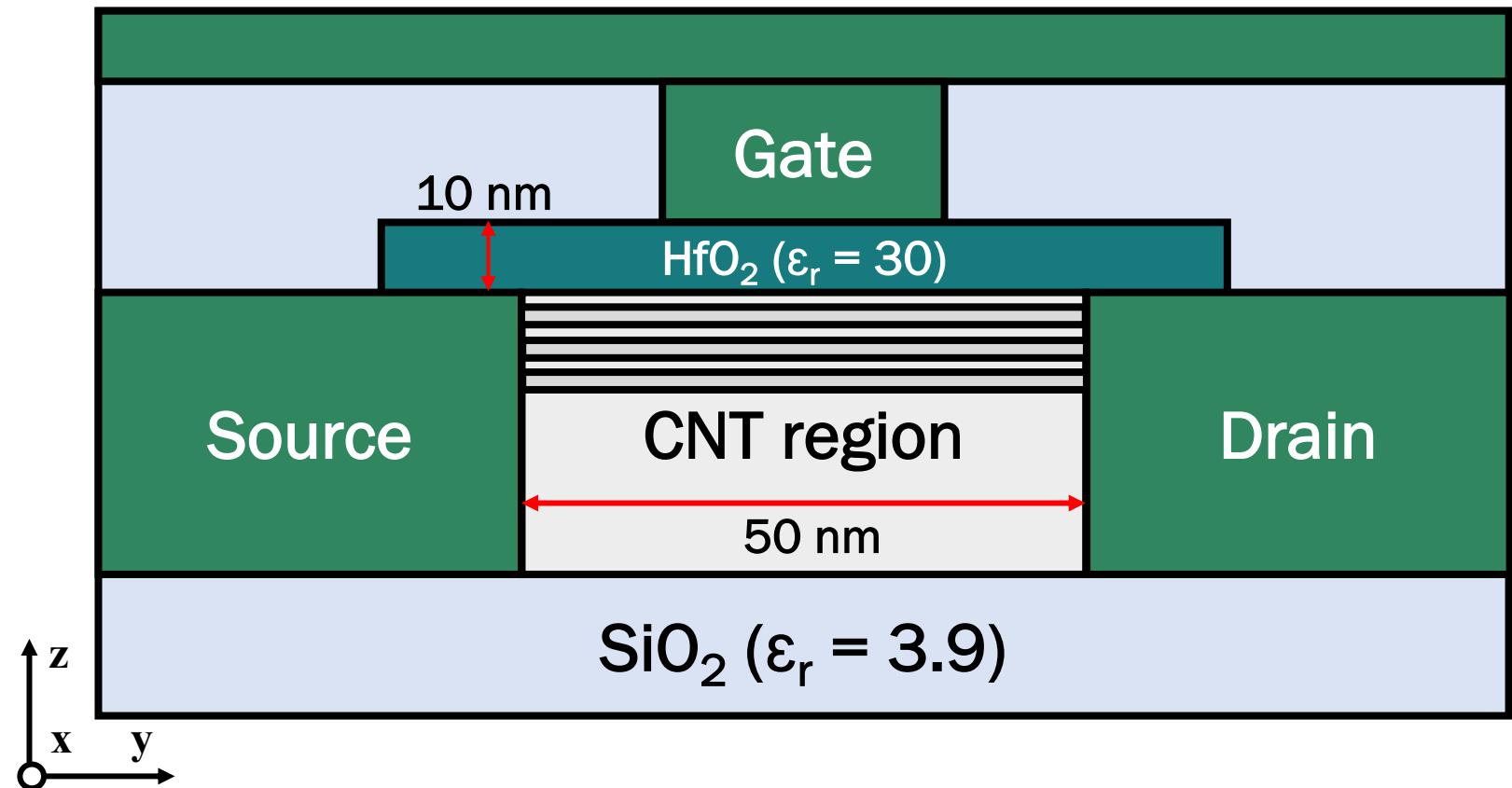
The Schrödinger equation is written for each individual transport channel

V is the electrostatic potential satisfying the Poisson equation

CNT-based FET: 3D Full-Wave simulation + Schrödinger – Poisson eqs. coupling

- Substrate – SiO_2
- Oxide – HfO_2
- Contacts – Gold

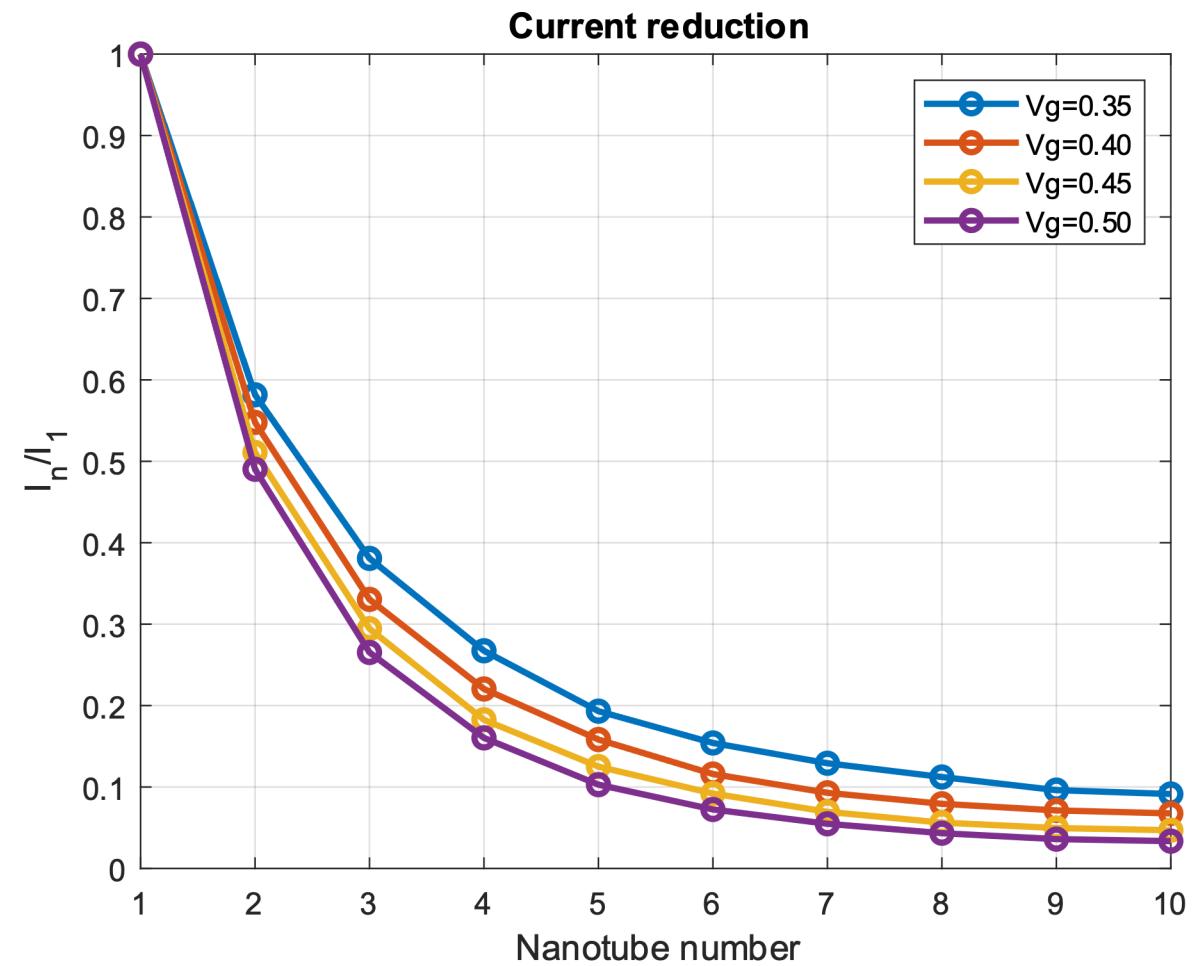
CNT (16,0)
• $R = 0.63 \text{ nm}$
• $L = 50 \text{ nm}$
• Spacing 0.50 nm



Results – Current reduction

2nd CNT 50–40% less
5th CNT 90–80% less

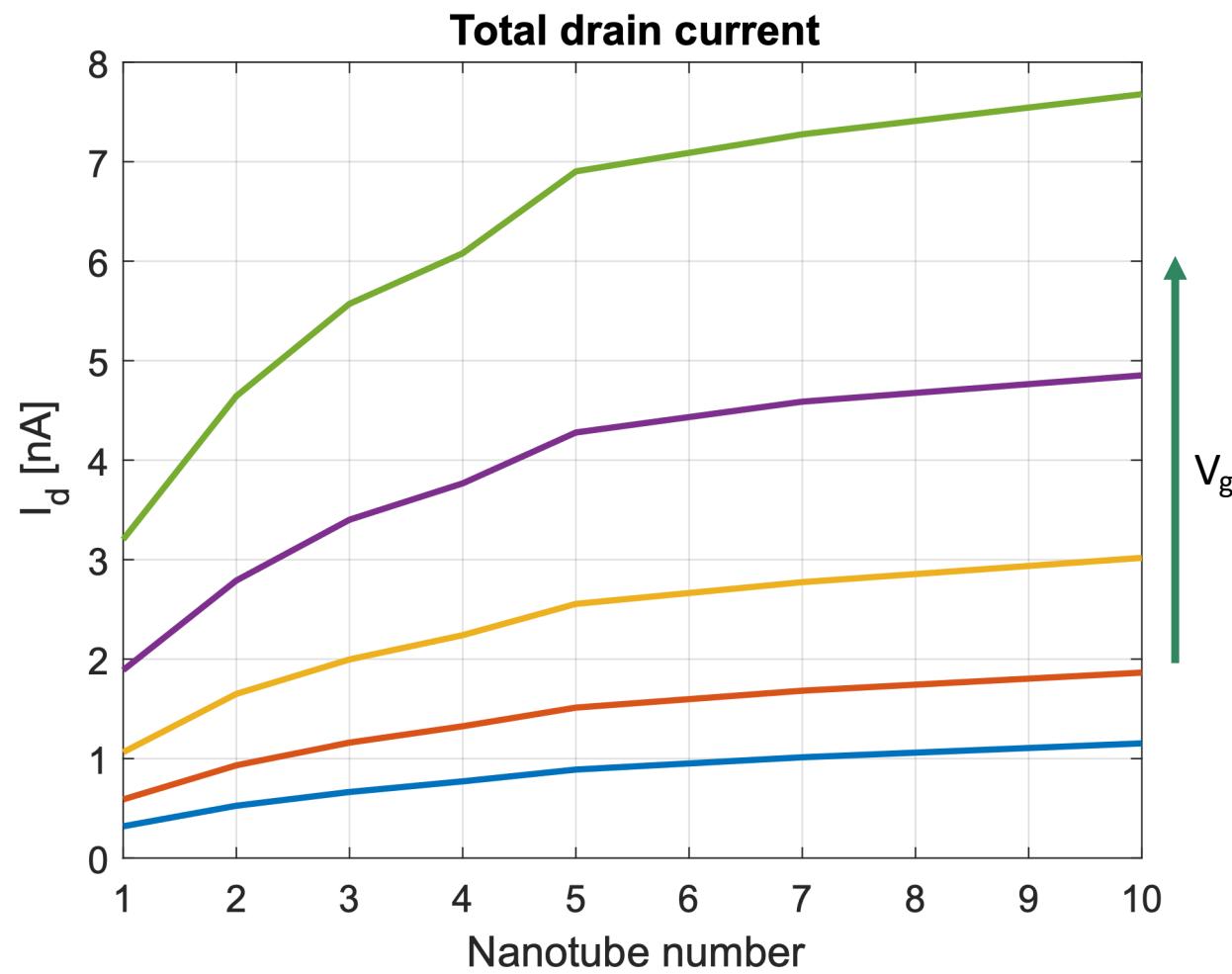
I_n Current in the nth CNT
 I_1 Current in the 1st CNT



Results – Current saturation

- Distance from the gate
- Shielding from above CNTs

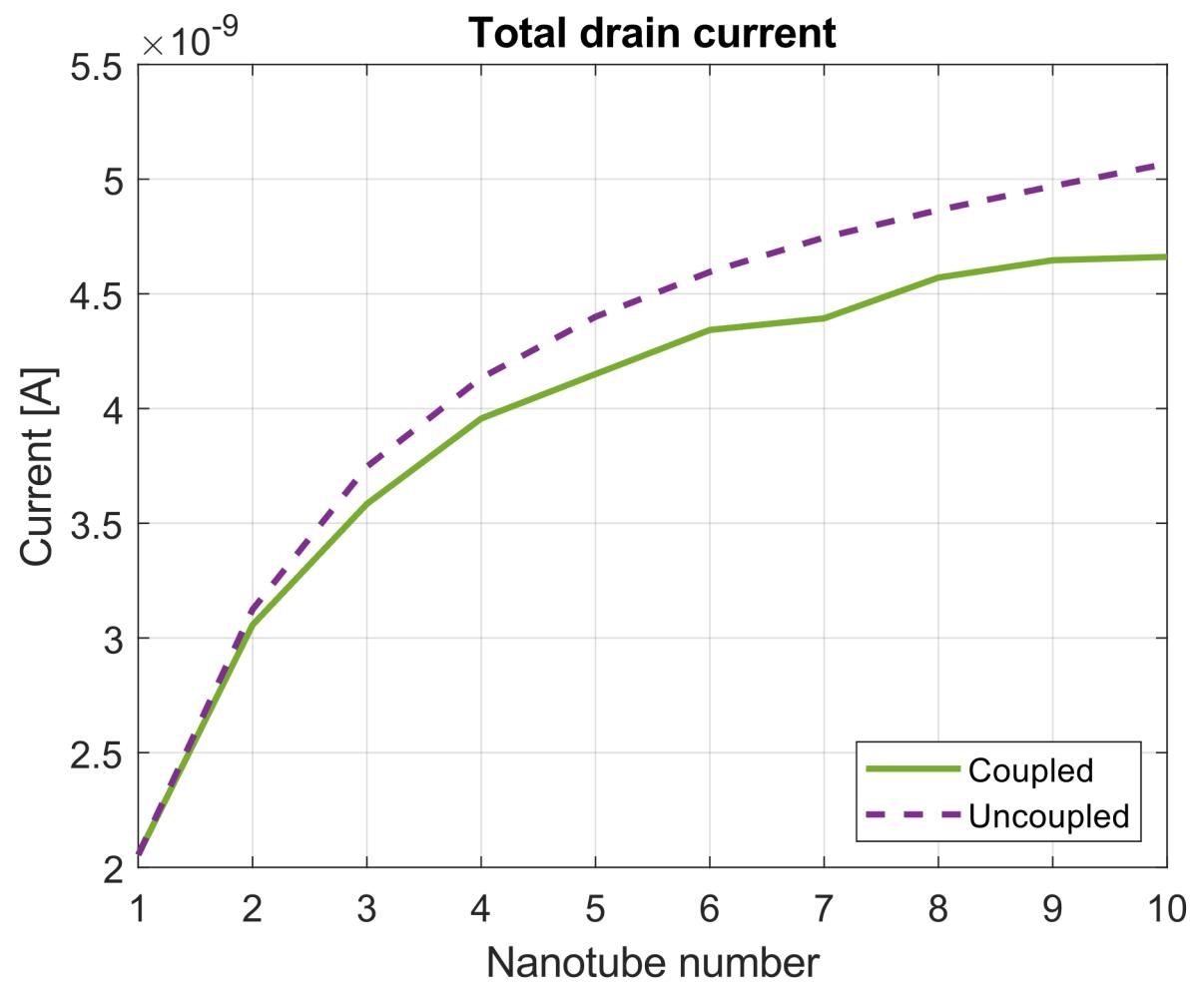
CNTs 1-5 more than 80% of I_{tot}



Results – Coupling

5 CNT 6% reduction
10 CNT 9% reduction

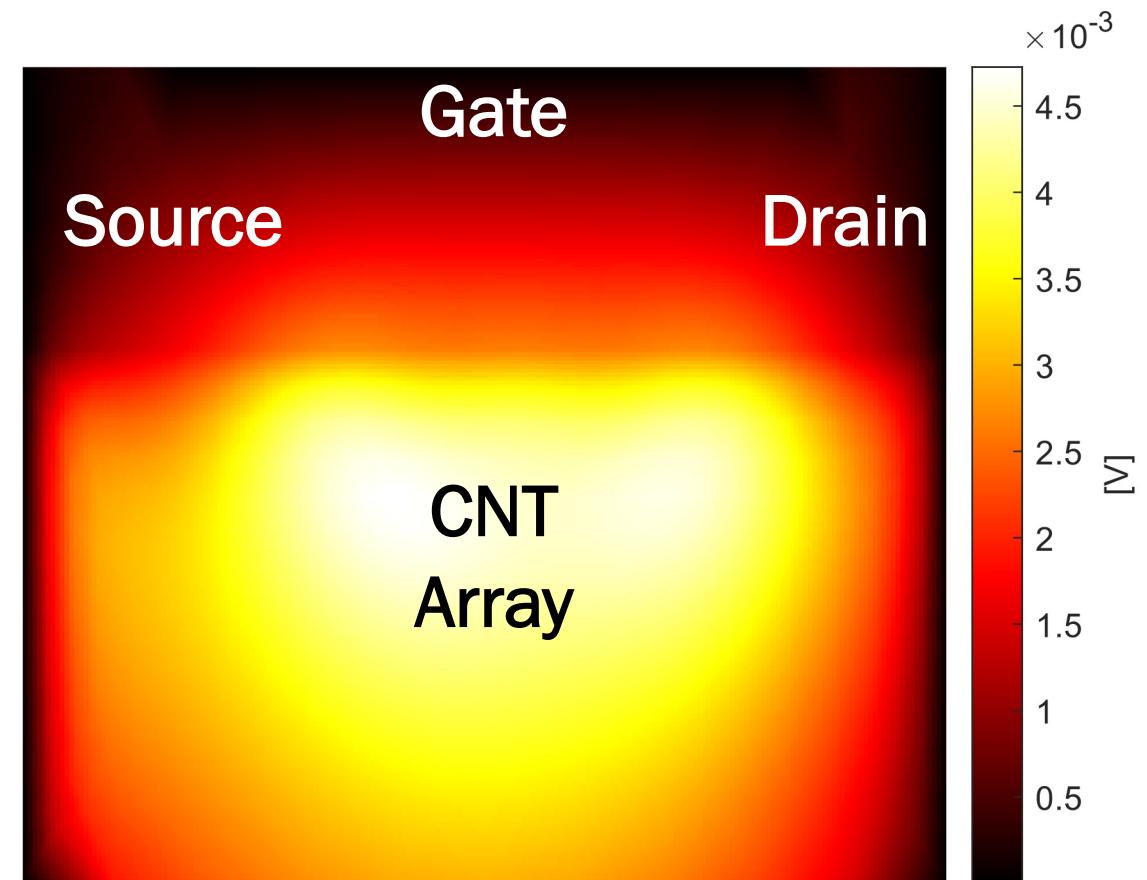
Uncoupled → One row at the time
Coupled → All rows at the same time



Results – Potential

Variation of the channel potential due to carriers in the CNT array

Shield Effect



Future works

- Presence of metallic nanotubes
- Hamiltonian terms for charge correlation
- Mechanics modes analysis
- RF parameters extraction

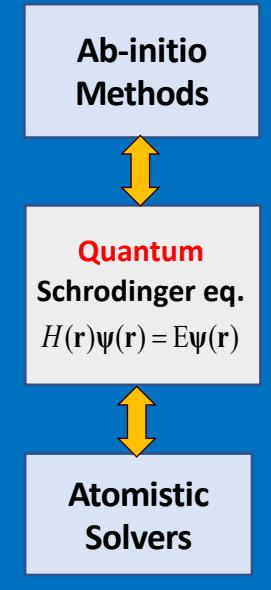
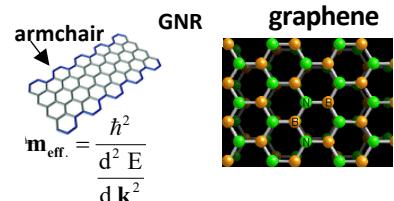
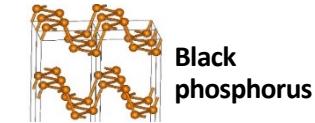
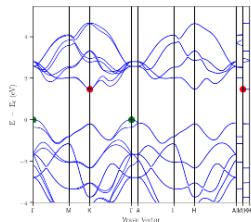
Examples

- Graphene Antenna
- MoS₂ FET
- Schrödinger – Poisson eqs.: CNT FET
- **Dirac – Maxwell eqs.: Ballistic Ratchet effect on graphene**

KEY DEVELOPMENT

- Bridging from **atomistic** to **continuum** level
- Interfacing **mathematical models** (PDEs)

ATOMISTIC LEVEL



THE BRIDGE

$$\begin{aligned}\varepsilon(\omega, \mathbf{k}) \\ \mu(\omega, \mathbf{k}) \\ \sigma(\omega, \mathbf{k}) \\ m_{\text{effective}}\end{aligned}$$

$$\mathbf{J}(\mathbf{r}) = \int d^2 r' \sigma_{HO}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')$$

$$\sigma_{HO} = \sigma_{\text{Hafnium Oxide}}$$

constitutive
eqs./relations

FULL-WAVE SOLVERS

Quantum: Schr./Dirac

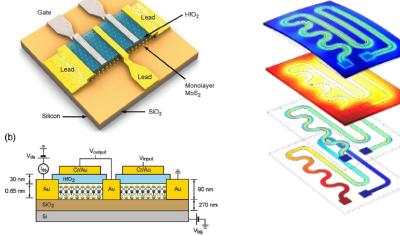
$$H(\mathbf{r}, t)\psi(\mathbf{r}, t) = i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t)$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}(\mathbf{r}, t)$$

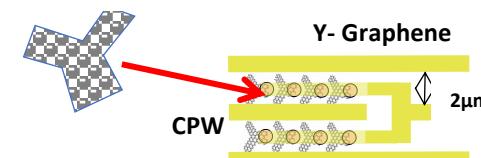
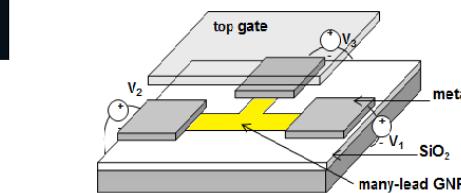
Electromagnetic:
Maxwell

MoS₂ FET



Devices
Systems
Interfaces

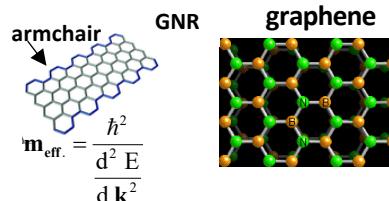
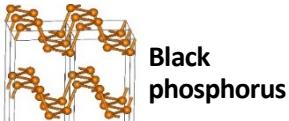
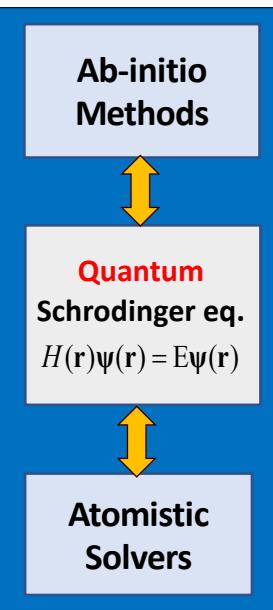
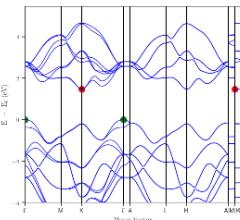
CONTINUUM LEVEL



Luca Pierantoni

Interfacing Physics ---> PDEs equations Systems

ATOMISTIC LEVEL



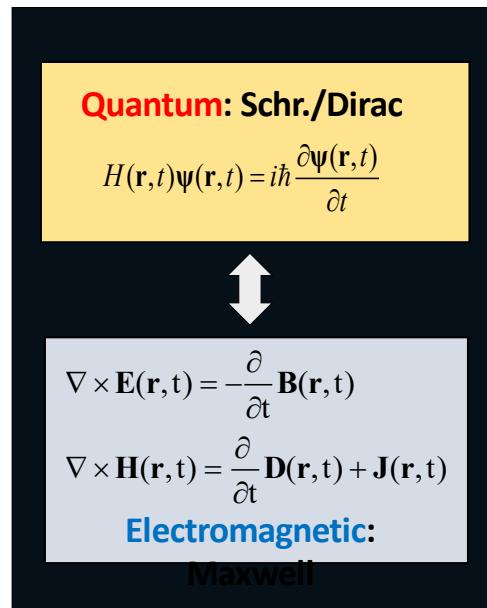
$$\begin{aligned} &\varepsilon(\omega, \mathbf{k}) \\ &\mu(\omega, \mathbf{k}) \\ &\sigma(\omega, \mathbf{k}) \\ &m_{\text{effective}} \end{aligned}$$

$$\mathbf{J}(\mathbf{r}) = \int d^2 r' \sigma_{HO}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')$$

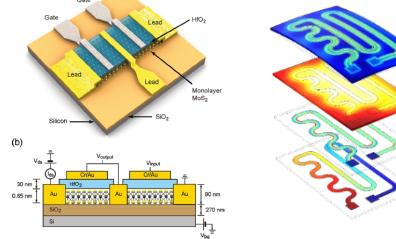
$\sigma_{HO} = \sigma_{\text{Hafnium Oxide}}$

constitutive
eqs./relations

FULL-WAVE SOLVERS

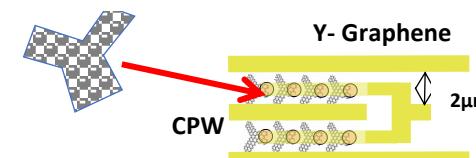
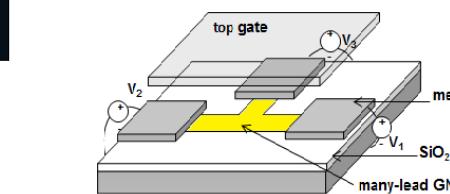


MoS₂ FET



Devices Systems Interfaces

CONTINUUM LEVEL

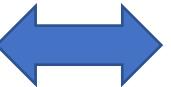


Luca Pierantoni

Electrodynamics and Quantum Transport : combining Maxwell < --- > Schrödinger/Dirac

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t)$$
$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}(\mathbf{r}, t)$$

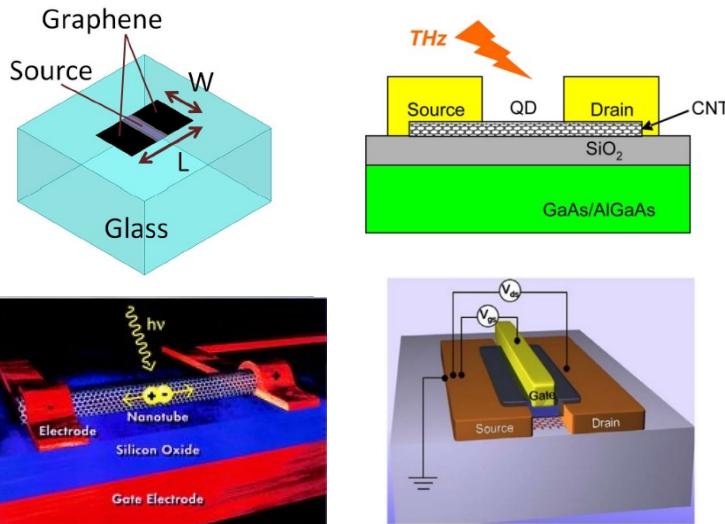
Electromagnetic: Maxwell



$$H(\mathbf{r}, t)\psi(\mathbf{r}, t) = i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t}$$

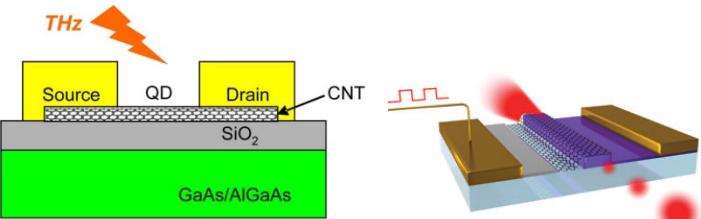
Quantum: Schrödinger/Dirac

- interaction particles-EM field - transient
- Quantum dots, quantum wells
- **Ballistic electronics**
- non-linear devices
- **Spintronics**
- **photodetectors**
- ...

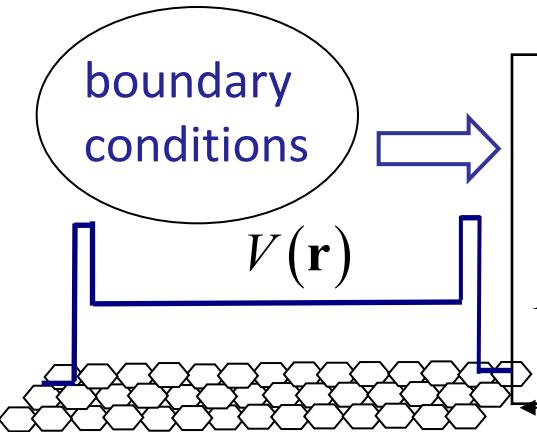


Home-made Software

the concept



ballistic transport
in the quantum device



Maxwell equations

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t)$$
$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}(\mathbf{r}, t)$$
$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t)$$
$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0$$

EM field

$t = t_0 + 1$

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$t = t_0$

$$\underline{H}(\mathbf{r}, t)\underline{\psi}(\mathbf{r}, t) = i\hbar \frac{\partial \underline{\psi}(\mathbf{r}, t)}{\partial t}$$
$$H(\mathbf{r}, t)\psi(\mathbf{r}, t) = i\hbar \left(\frac{\partial}{\partial t} + \frac{ie}{\hbar} \varphi(\mathbf{r}, t) \right) \psi(\mathbf{r}, t)$$

Dirac equation

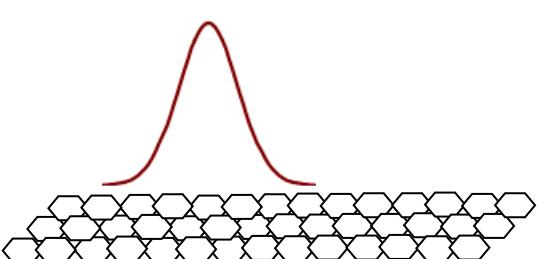
$$\underline{\psi}(\mathbf{r}, t)$$

wave function

$$\mathbf{J}(\mathbf{r}, t)$$

quantum-mechanical current

$$q|\underline{\psi}(\mathbf{r}, t)|^2$$



initial conditions

$$\underline{\psi}(\mathbf{r}, t=0)$$

Quantum-EM: Dirac Equation in the presence of an EM field

3D

$$i\hbar \frac{\partial \Psi_+}{\partial t} = v_F \boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} + |e| \mathbf{A}) \Psi_- + (mv_F^2 - |e|\phi) \Psi_+$$

$$i\hbar \frac{\partial \Psi_-}{\partial t} = v_F \boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} + |e| \mathbf{A}) \Psi_+ - (mv_F^2 + |e|\phi) \Psi_-$$

$$\Psi = \begin{bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} = \begin{bmatrix} \Psi_+ \\ \Psi_- \end{bmatrix} \quad \text{with} \quad \Psi_+ = \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} \quad \Psi_- = \begin{bmatrix} \psi_2 \\ \psi_3 \end{bmatrix}$$

2D

$$i\hbar \frac{\partial \Psi}{\partial t} = \boldsymbol{\sigma}_{xy} \cdot \hat{\mathbf{p}}_{xy} v_F \Psi + mv_F^2 \sigma_z \Psi$$

$$v_F \approx \frac{c}{300}$$

$$\boldsymbol{\sigma} = \sigma_x \hat{x} + \sigma_y \hat{y} + \sigma_z \hat{z} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \hat{x} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \hat{y} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \hat{z} = \boldsymbol{\sigma}_{xy} + \sigma_z \hat{z}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\mathbf{p}} = p_x \hat{x} + p_y \hat{y} + p_z \hat{z} = -i\hbar \nabla = (-i\hbar \partial_x) \hat{x} + (-i\hbar \partial_y) \hat{y} + (-i\hbar \partial_z) \hat{z} = \hat{\mathbf{p}}_{xy} + p_z \hat{z}$$

Dirac Equation – Maxwell Equations

Graphene devices: ballistic transport, effective mass

modeling of discontinuities – absorbing boundary conditions

$$i\hbar \frac{\partial \psi^+}{\partial t} = \boldsymbol{\sigma}_{xy} \cdot (\hat{\mathbf{p}}_{xy} - q\mathbf{A}) v_F \psi^- + M(\mathbf{r}) v_F^2 \sigma_z \psi^+ \rightarrow$$

the mass term M
may models discontinuities

$$i\hbar \frac{\partial \psi^-}{\partial t} = \boldsymbol{\sigma}_{xy} \cdot (\hat{\mathbf{p}}_{xy} - q\mathbf{A}) v_F \psi^+ + M(\mathbf{r}) v_F^2 \sigma_z \psi^-$$

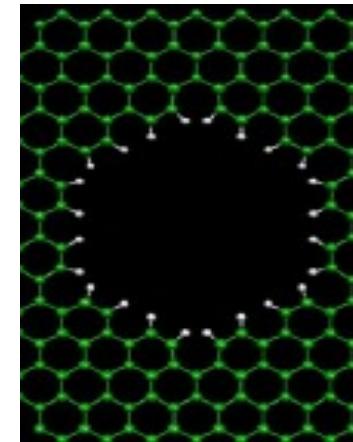
$$\boldsymbol{\sigma} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \hat{x} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \hat{y} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \hat{z} = \boldsymbol{\sigma}_{xy} + \sigma_z \hat{z}$$

$$\psi = \begin{bmatrix} \psi^+ \\ \psi^- \end{bmatrix}$$

$$\hat{\mathbf{p}} = -i\hbar\nabla = \hat{\mathbf{p}}_{xy} + p_z \hat{z}$$

$$\mathbf{J} = v_F \psi^* \hat{\sigma} \psi$$

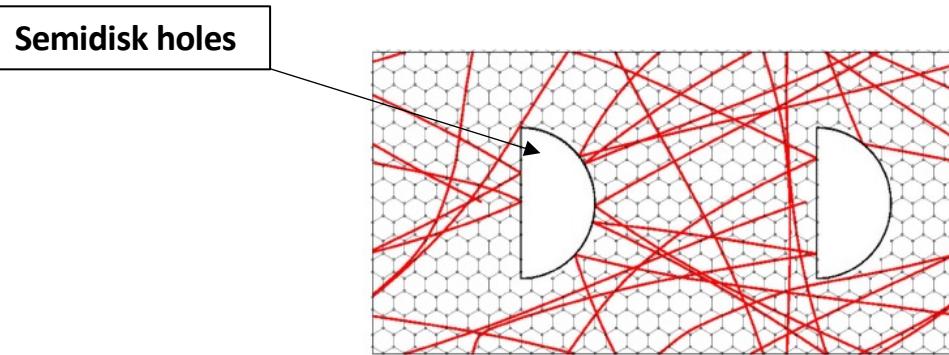
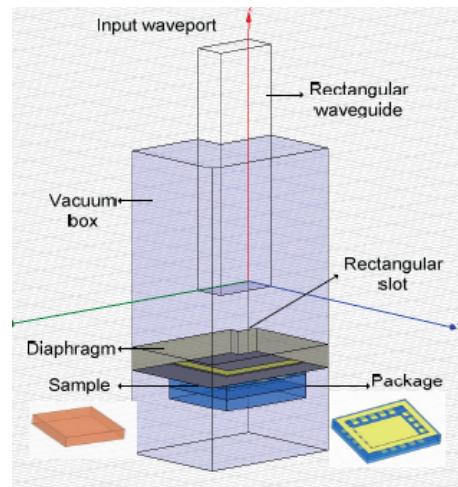
$$\hat{\sigma} = \begin{bmatrix} \sigma_x & \sigma_y \end{bmatrix}$$



obstacles/discontinuities can be analyzed by means of the effective mass concept

Example: Dirac + Maxwell in Ballistic Ratchet effect in antidot patterned on graphene

... a collective motion of particles in a preferential direction, due to spatially-asymmetric perturbations



L.Ermann and D.L.Shevelyansky: Relativistic graphene ratchet

... under microwave linear polarized radiation, the effect was observed in a **high mobility** two-dimensional electron gas based on **AlGaAs/GaAs** heterojunction, with periodic array of artificial semi-discs shaped obstacles



GOAL: graphene in place of semiconductor

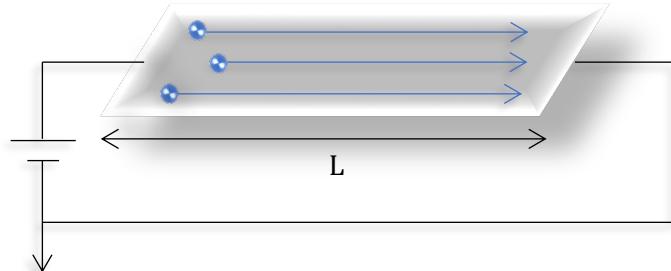
(*) D. Medhat, A. Takacs, H. Aubert, J. C. Portal, **Comparative Analysis of Different Techniques for Controlling Ratchet Effect in a Periodic Array of Asymmetric Antidots**, [Micr.Conference, \(2009\). APMC 2009. Asia Pacific](#)

S. Bellucci, L. Pierantoni, D. Mencarelli, **"Ballistic Ratchet effect on patterned graphene"**, *Journal of Integrated Ferroelectrics*, Vol. 176, Issue 1, pp. 28-36, Dec. 2016, DOI: <http://dx.doi.org/10.1080/10584587.2016.1185883>

Transport: Ballistic vs Diffusive

$$\begin{aligned}\langle l_{graphene} \rangle &\approx 10^{-6} \text{ [m]} \\ \langle v_{graphene} \rangle &\approx 10^6 \text{ [m / s]} \\ \langle \tau_{graphene} \rangle &\approx 10^{-12} \text{ [s]}\end{aligned}$$

BALLISTIC TRANSPORT

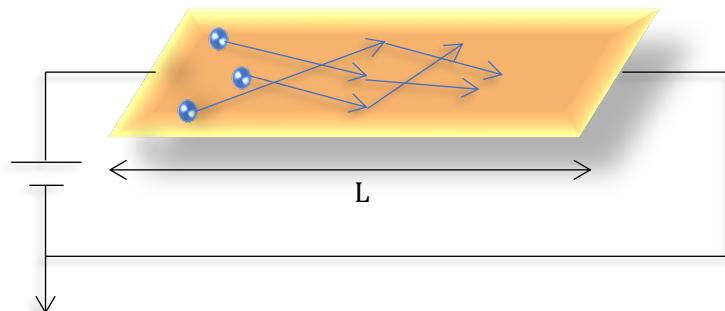


Ballistic Transport = Electrons travel **without scattering** from injected contact to absorbing contacts

weak recombination with phonons

$$\begin{aligned}\langle l_{cu} \rangle &\approx 10^{-10} \text{ [m]} \\ \langle v_{cu} \rangle &\approx 10^5 \text{ [m / s]} \\ \langle \tau_{cu} \rangle &\approx 10^{-15} \text{ [s]}\end{aligned}$$

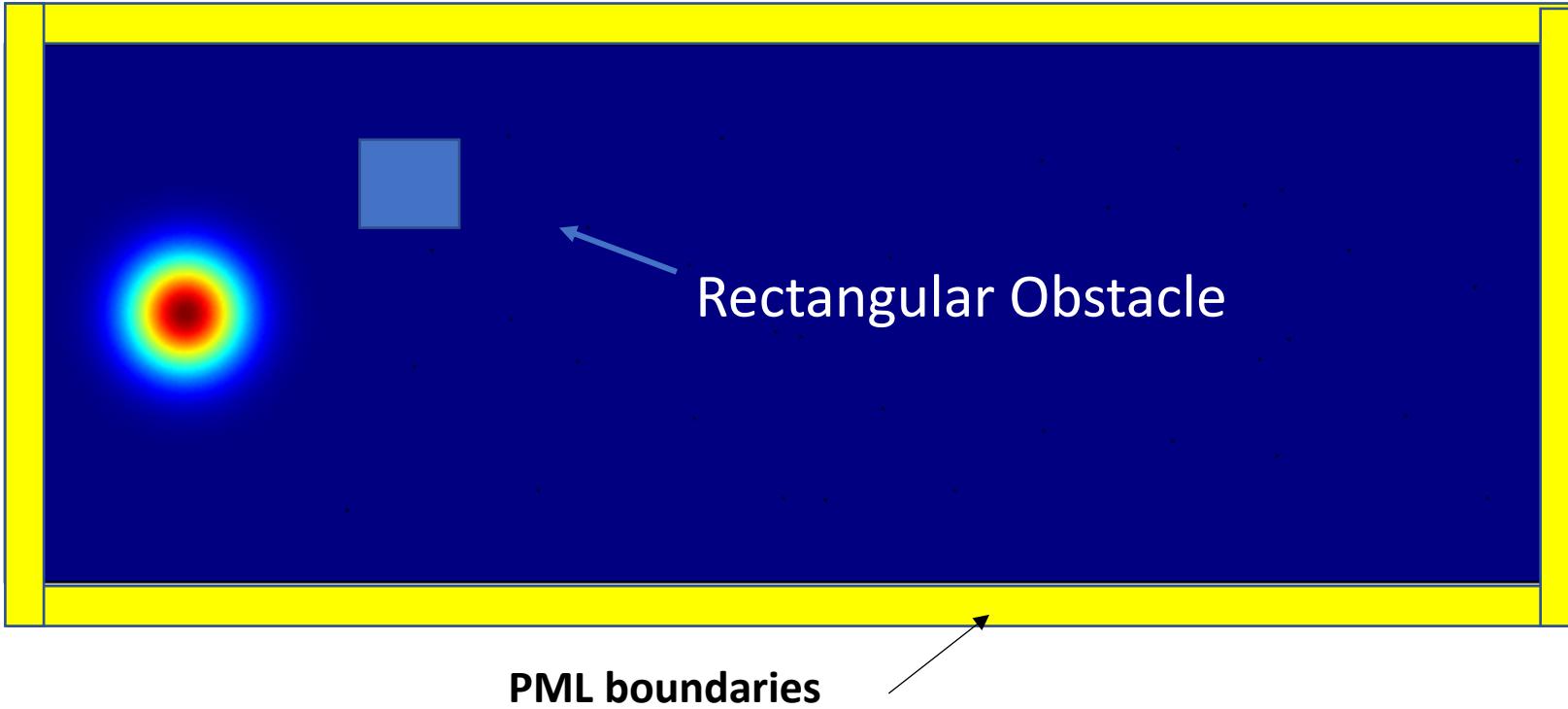
NON-BALLISTIC (DIFFUSIVE) TRANSPORT



Diffusive Transport = Electrons undergo a **random walk** as they go from left to right contact

- the average distance between collisions is called the **mean free path (λ)**
- The diffusive transit time will be much longer than the ballistic transit time

scattering/diffraction by obstacles



$$\Delta L_{\min} = 1 \text{ nm}$$
$$\Delta t = 0.5 \text{ fs}$$

$$T_{\max} = 100 \text{ fs}$$

$$n_s(t_0) = 10^{11} \text{ cm}^{-2}$$

$$E_0 = 0.5 \text{ eV}$$

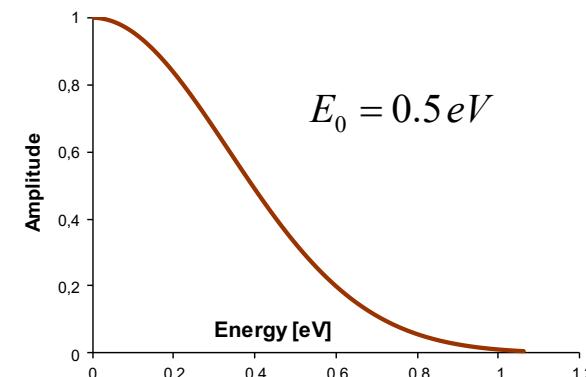
$$\Delta_{PML} = 5 \text{ nm}$$

$$\psi = \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} \quad \begin{cases} \psi_+ = f(x, y; t = 0) \\ \psi_- = 0 \end{cases}$$

$$f(x, y; t = 0) = \psi_N \left\{ \exp\left(\frac{-(x - x_0)^2}{2\sigma_x^2}\right) \cdot \exp\left(\frac{-(y - y_0)^2}{2\sigma_z^2}\right) \right\}$$

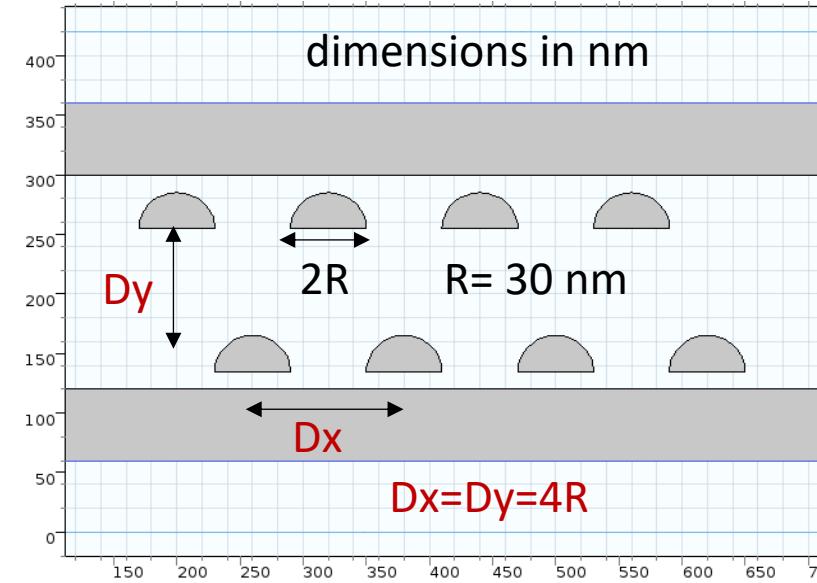
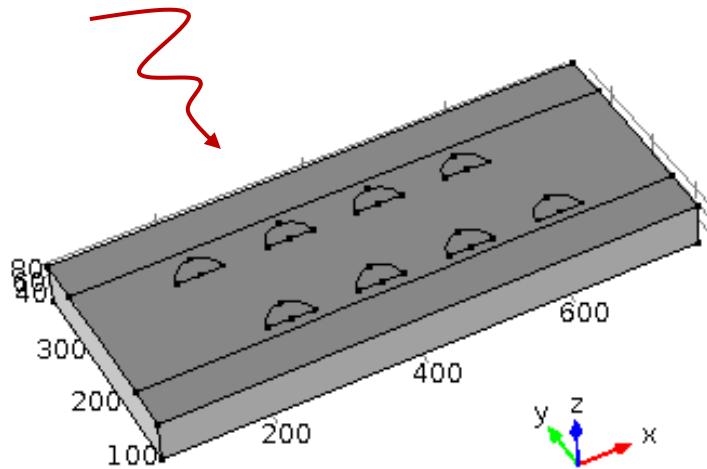
$$\int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = 1 \Rightarrow q \int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = Q_0 \Rightarrow \psi_N$$

$$n_s(t_0) \Leftrightarrow Q_0$$



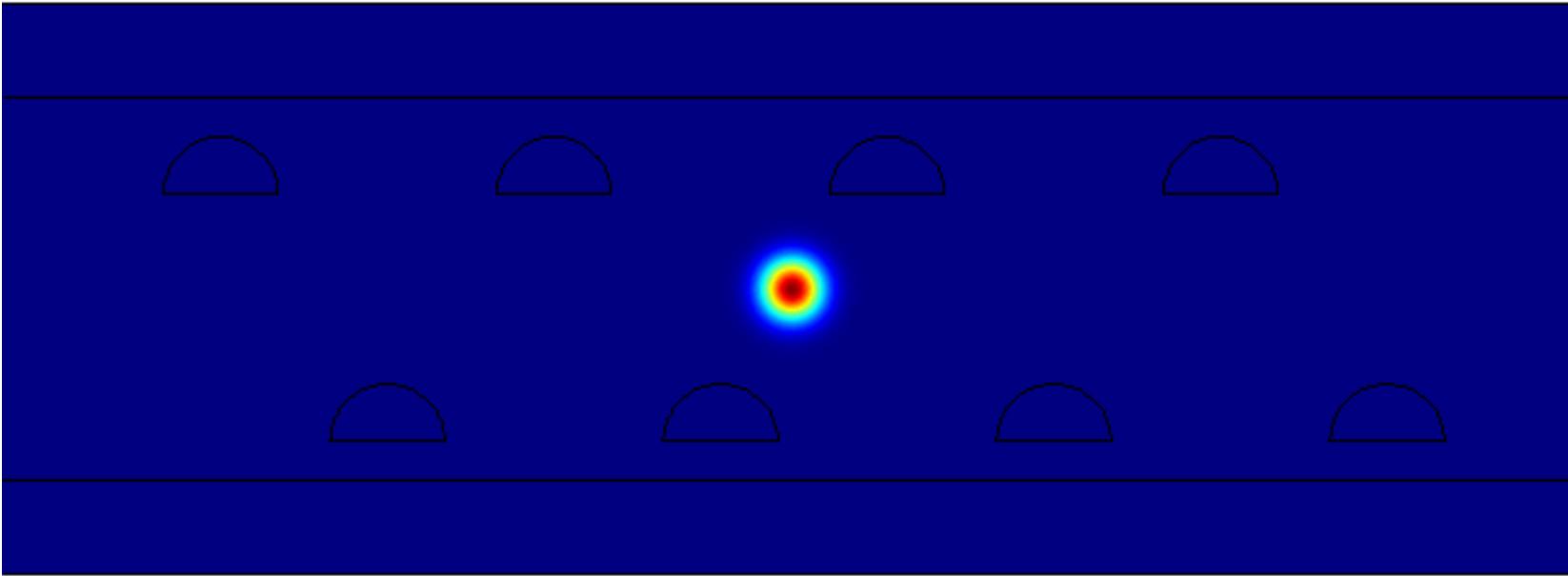
A plane wave couples with a graphene charge wavepacket

$$E_{ext} = E_0 \cos(\omega_0 t)$$



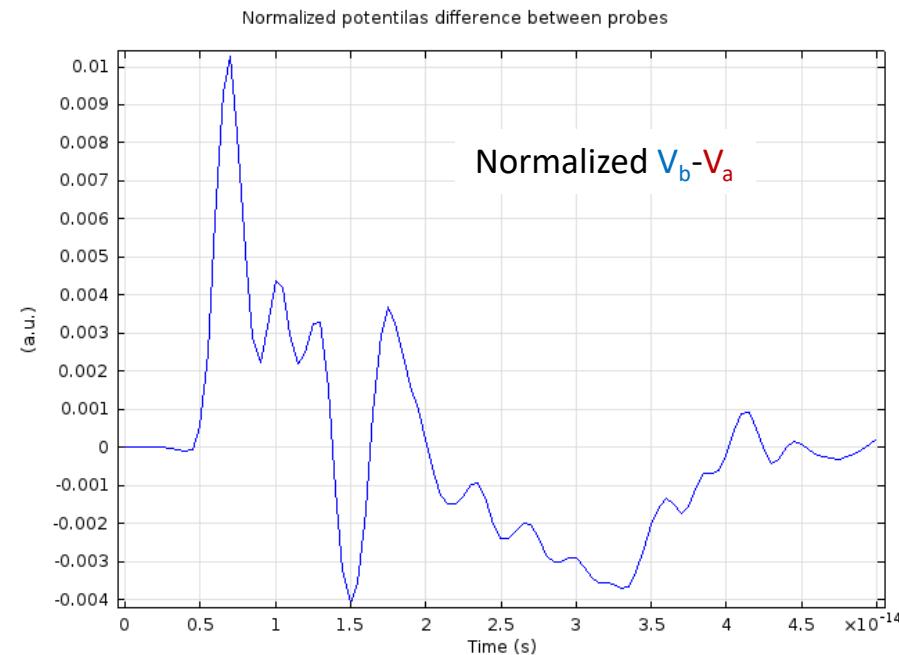
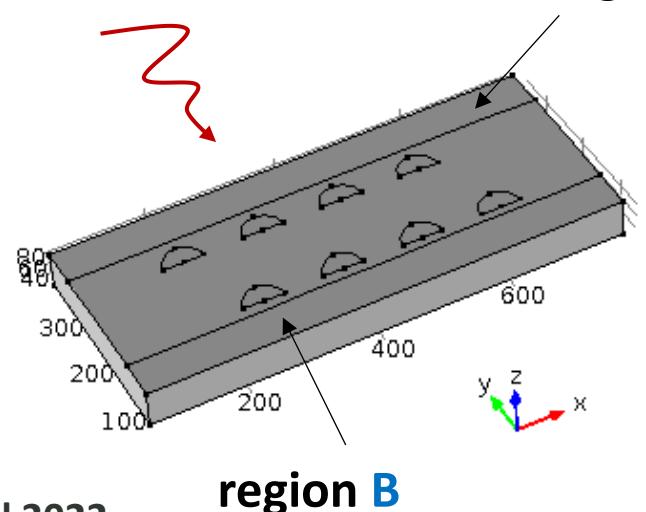
- a uniform external time-dependent **plane wave** is impinging on the graphene
- EM plane wave spectrum up to **THz**
- a **charge wavepacket** is set on graphene
- corresponding Energy: up to **0.5 eV**
- $Q(t=0)$ corresponds to $n_s(t_0) = 10^{11} \text{ cm}^{-2}$

charge diffraction by ELLIPTIC obstacles

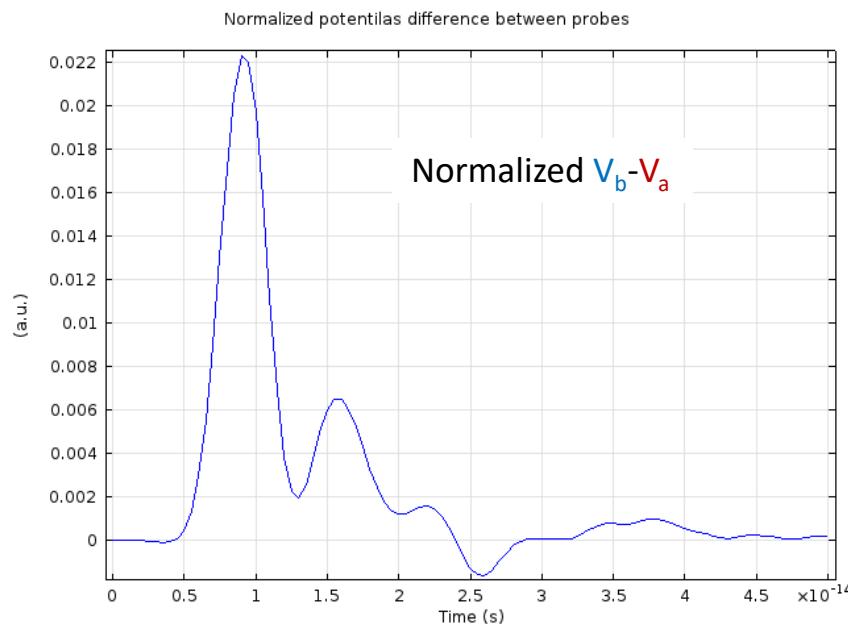
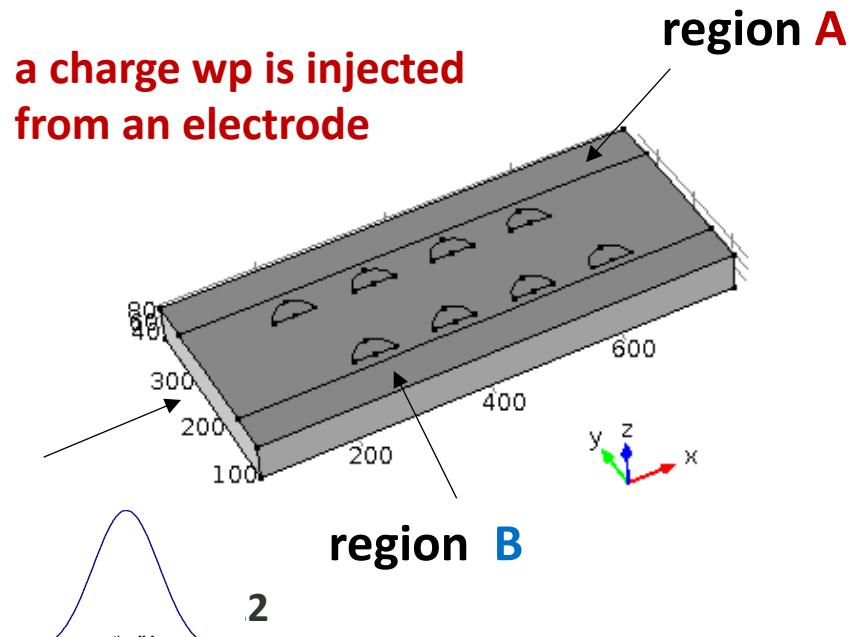
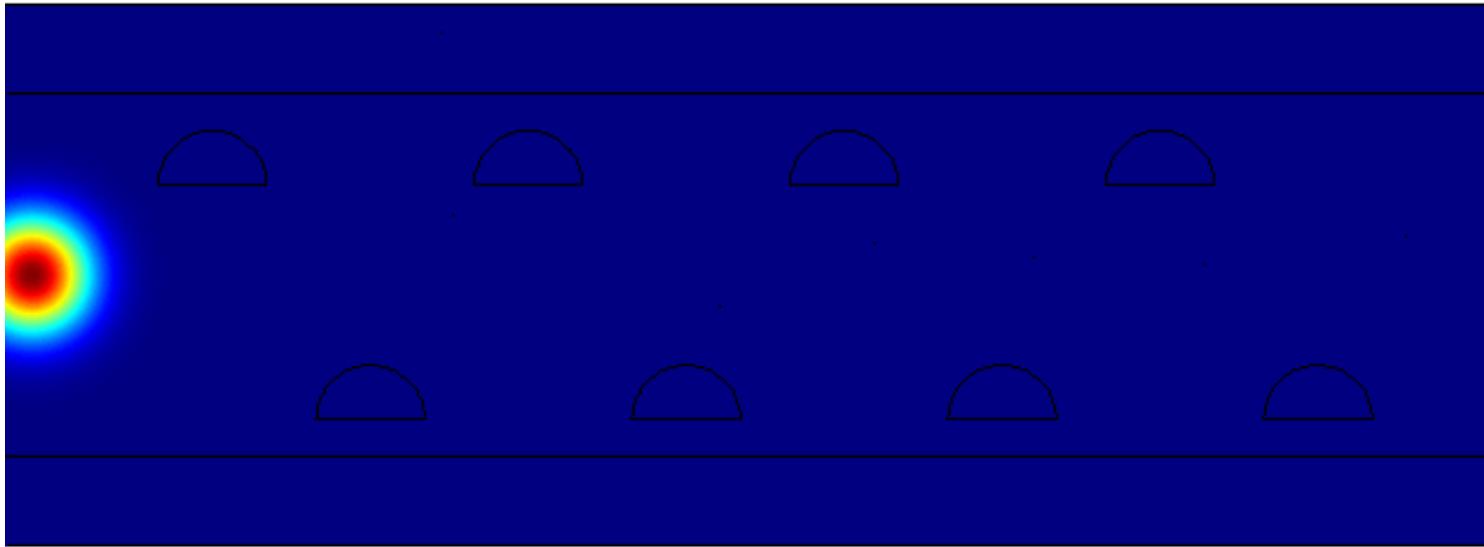


$$E_{ext} = E_0 \cos(\omega_0 t)$$

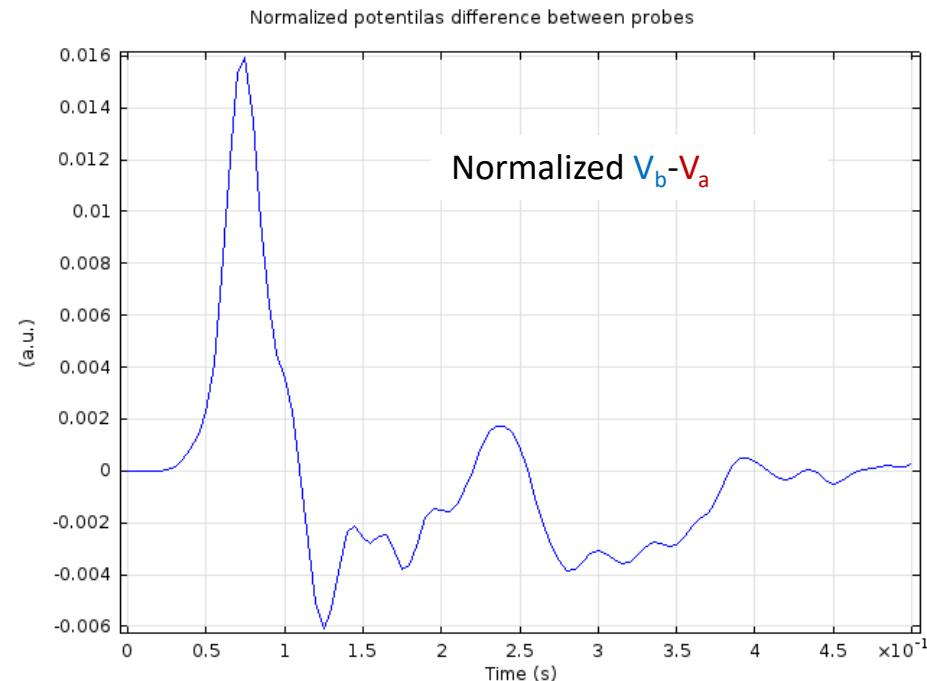
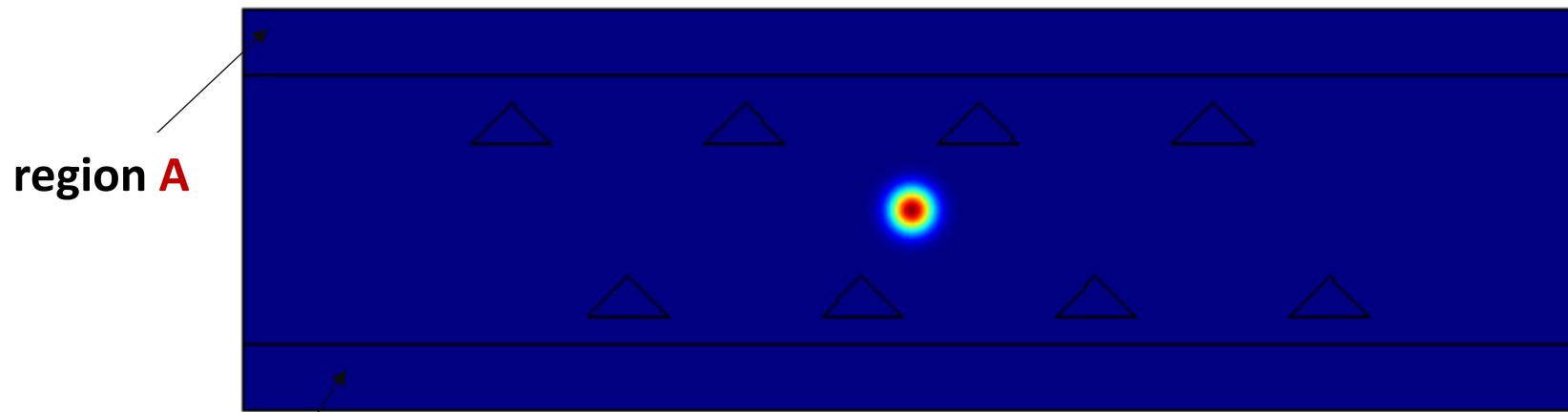
Software:
originanally
written in Fortran



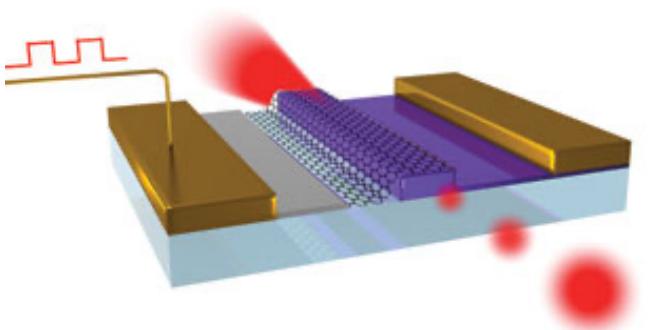
- coupling bewteen EM propagation and charge transport
- charge diffraction by obstacles



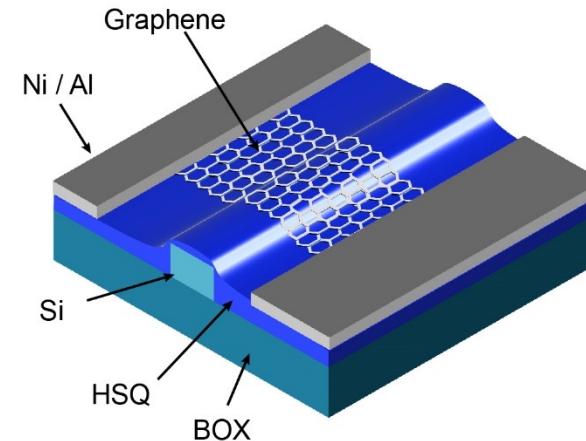
charge diffraction by TRIANGULAR obstacles



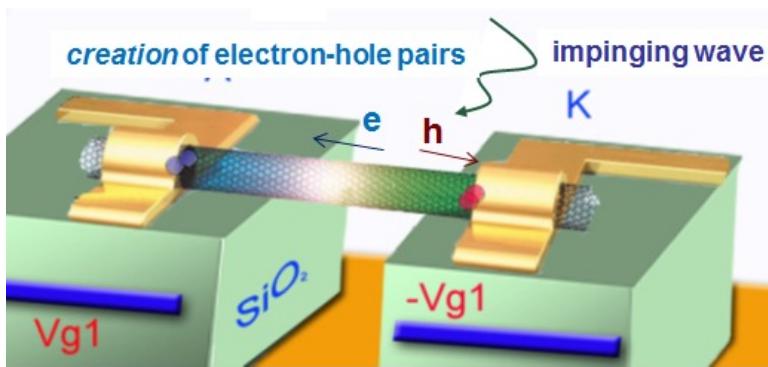
Application of the models/tools in the area of smart material-based Photonics



graphene optical modulator



graphene infrared detector



CNT photodetector

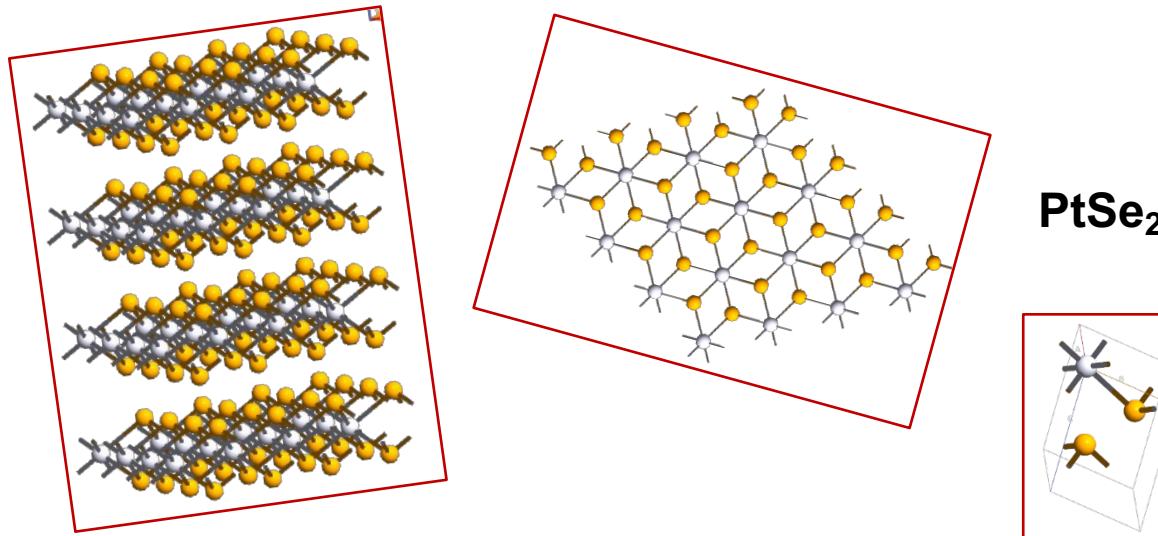
D. Mencarelli, L. Pierantoni, T. Rozzi, “Optical Absorption of Carbon Nanotube Diodes: Strength of the Electronic Transitions and Sensitivity to the Electric Field Polarization”, Journal of Applied Physics, vol. 103, Issue 6, pp.0631-03, March 2008, DOI: [10.1063/1.2890392](https://doi.org/10.1063/1.2890392)

THANK YOU !!!

Back up slides

Platinum diselenide (PtSe_2)-based devices

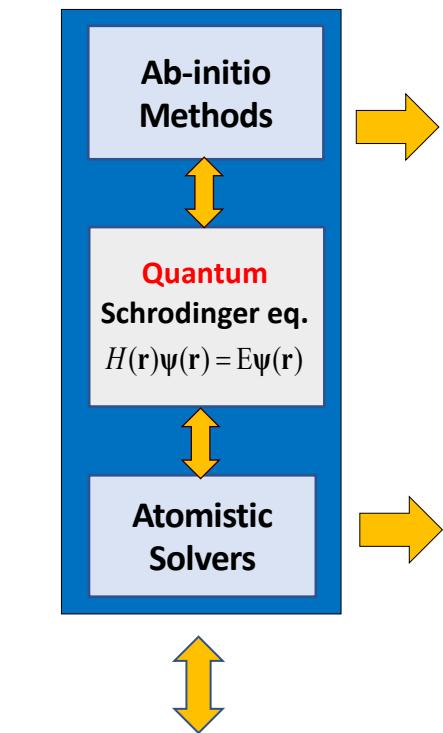
- Graphene has **high mobility** but **little bandgap**
- MoS_2 has **sizable bandgap** but **low mobility**
- Black phosphorus has **high mobility** and **sizable bandgap**, but is **unstable**
- PtSe_2 has **high mobility**, **sizable bandgap**
- CMOS compatible** with typical thin-film transistor processes
- is semimetallic, with **low-resistance contacts**—a challenge



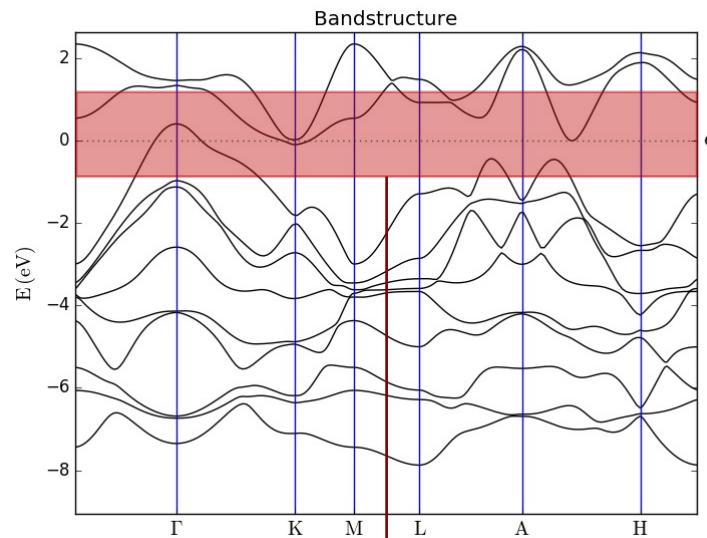
Platinum diselenide is a transition metal dichalcogenide with a layered structure. It has an Hexagonal unit cell with $a = b = 0.375 \text{ nm}$, $c = 0.506 \text{ nm}$ and $\alpha = \beta = 90^\circ$, $\gamma = 120^\circ$.

STEP I: ATOMISTIC SIMULATIONS

ATOMISTIC LEVEL



PtSe₂ is a metal or semiconductor depending on thickness:
bulk material shows a metallic behaviour.

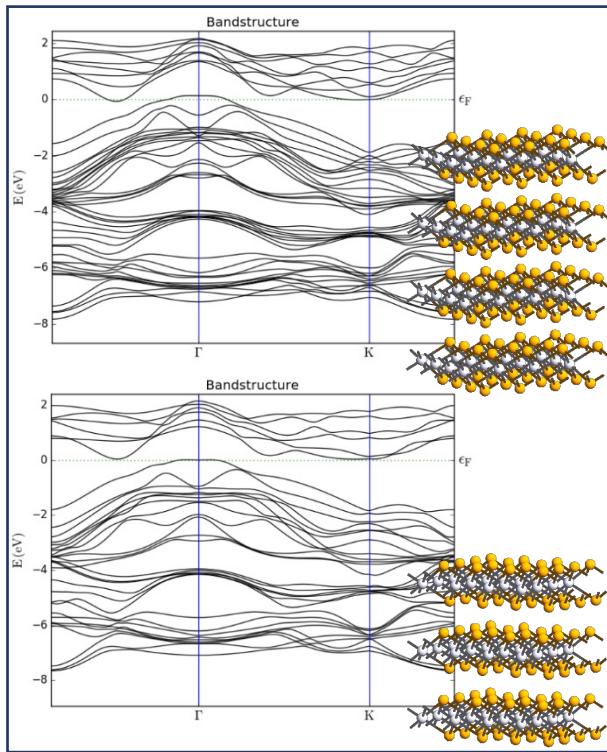


No Energy band-gap

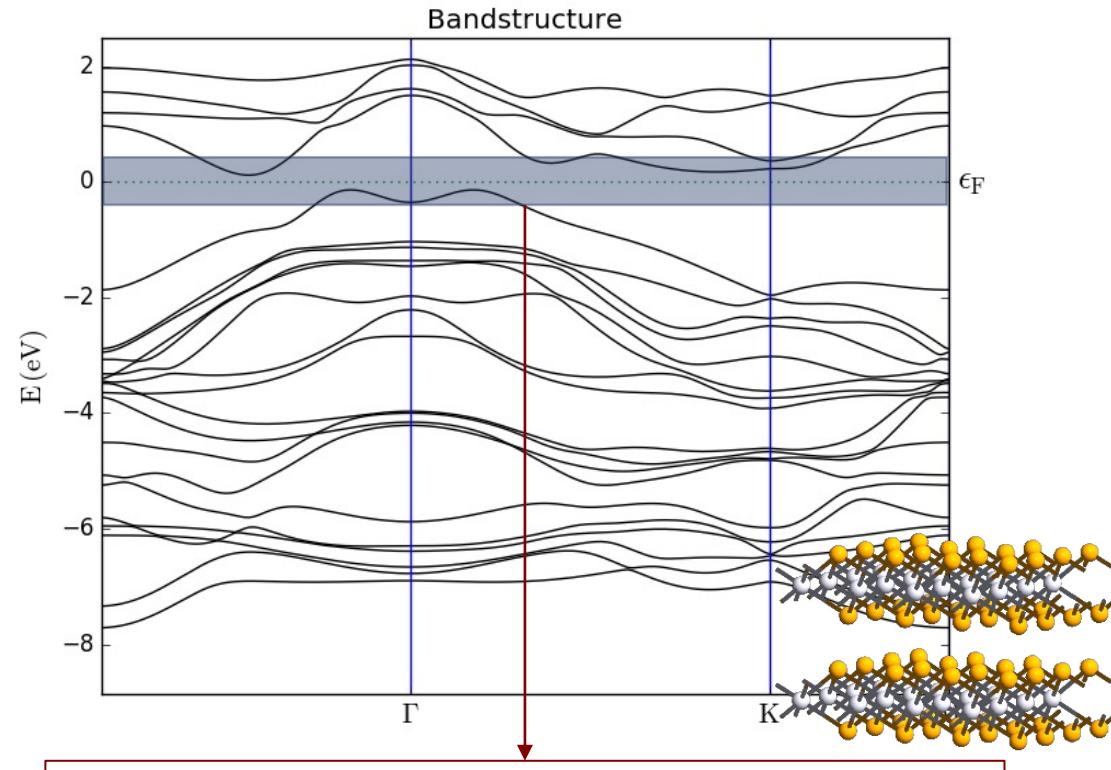
bulk material

Luca Pierantoni

Band-structure vs. number of layers



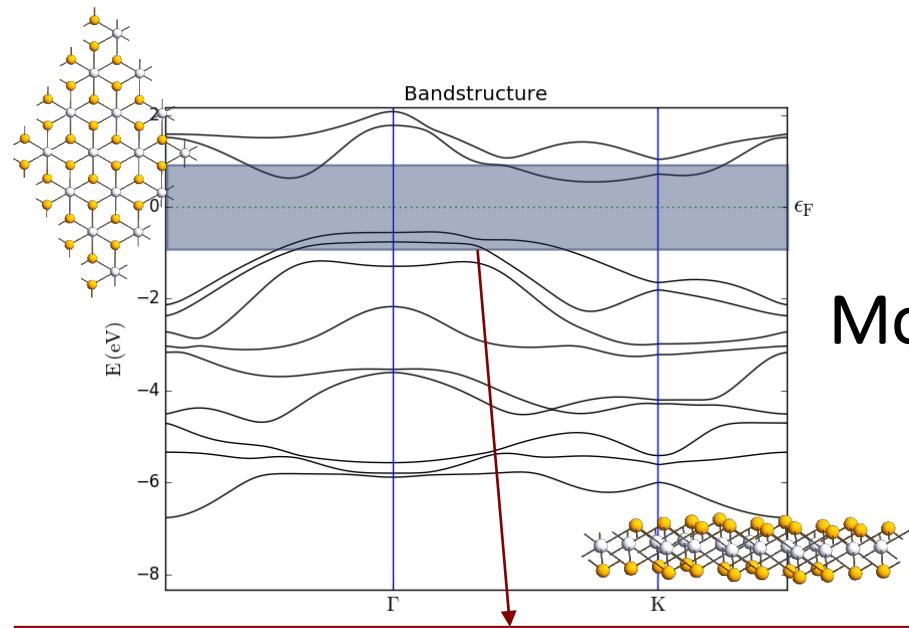
Four- and Three-Layer structure:
No Energy Band-Gap



Two-Layer structure: Energy Band-Gap of 0.31 eV, in
agreement with the experimental results presented in [1] (0.35
eV)

[1]: High-Electron-Mobility and Air-Stable 2D Layered PtSe₂ FETs, Zhao et. al., *Adv. Mater.* 2017, 29, 1604230

Band-structure vs. data in literature/experimental results



Monolayer structure

Monolayer structure: Energy Band-Gap of **1.147 eV**, in
agreement with the experimental results presented
in [1] (1.14 eV) and with the numerical calculations
done in [2] and [3] (1.2, 1.17 eV)

[1]: Direct observation of spin-layer locking by local Rashba effect in monolayer semiconducting PtSe_2 film, W. Yao, *Nature Communications*. 2017, 8, 14216

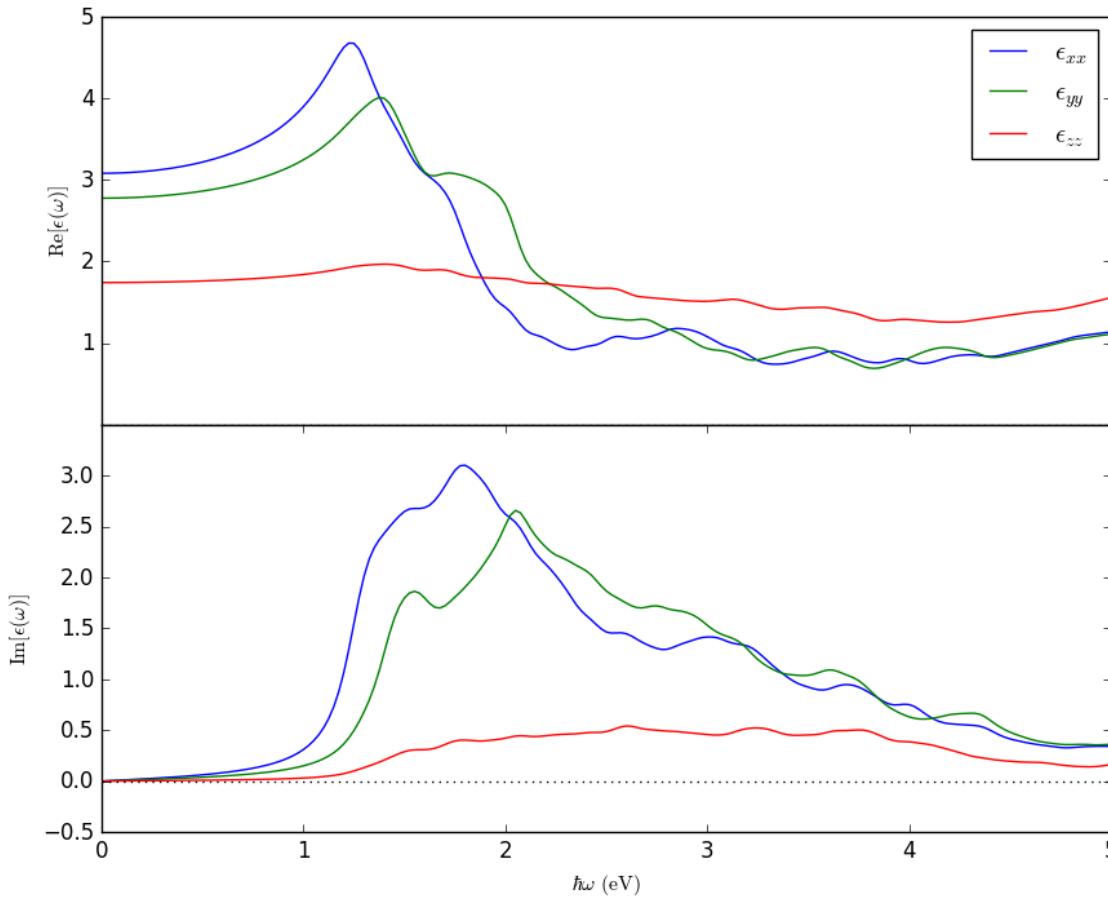
[2]: Monolayer PtSe_2 , a New Semiconducting Transition-Metal-Dichalcogenide, Epitaxially Grown by Direct Selenization of Pt, Y. Wang, *Nano Lett.*, 2015, 15, 4013-4018

[3]: High-Electron-Mobility and Air-Stable 2D Layered PtSe_2 FETs, Zhao et. al., *Adv. Mater.* 2017, 29, 1604230

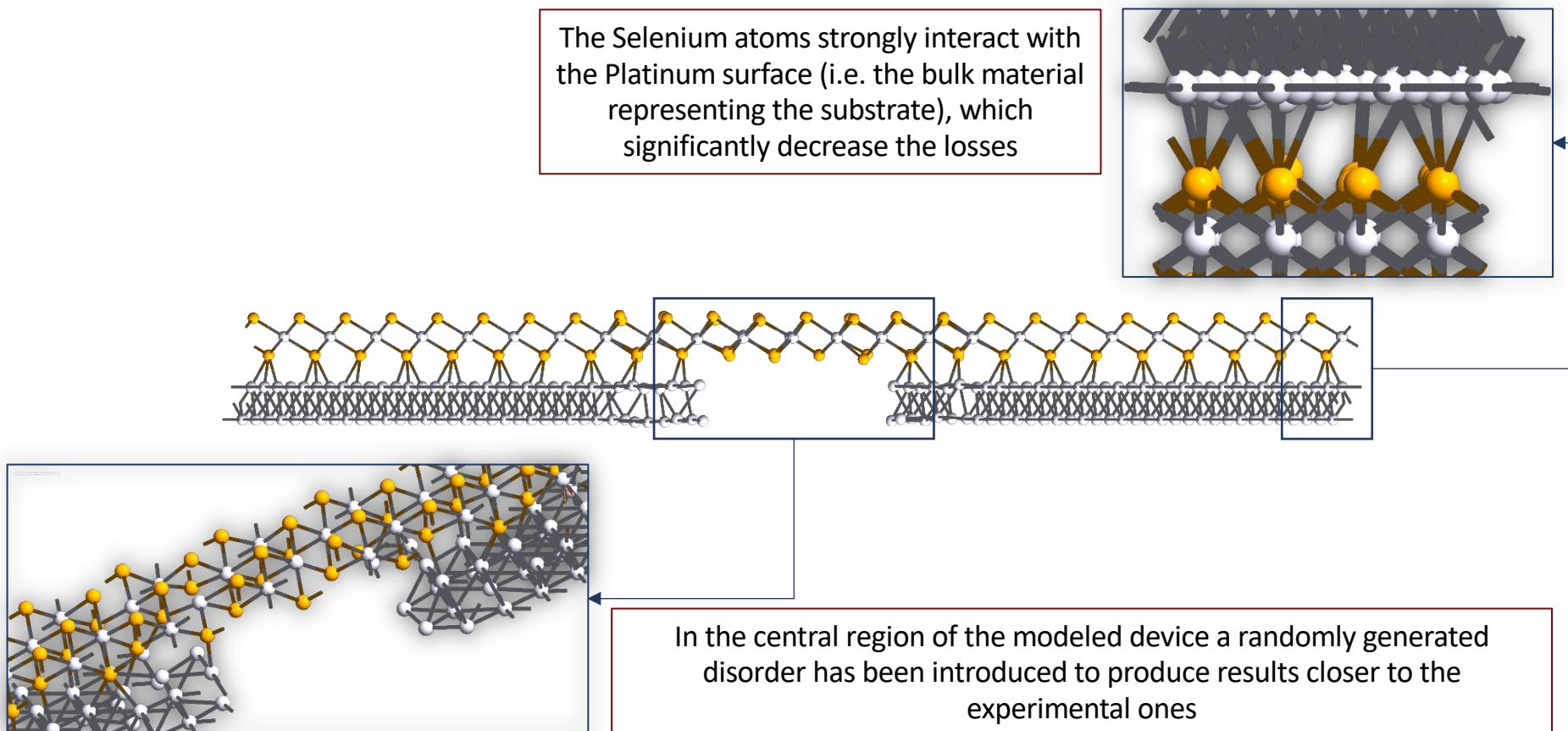
STEP II: obtaining the constitutive relations



Relative Permittivity $\epsilon_r(\omega)$

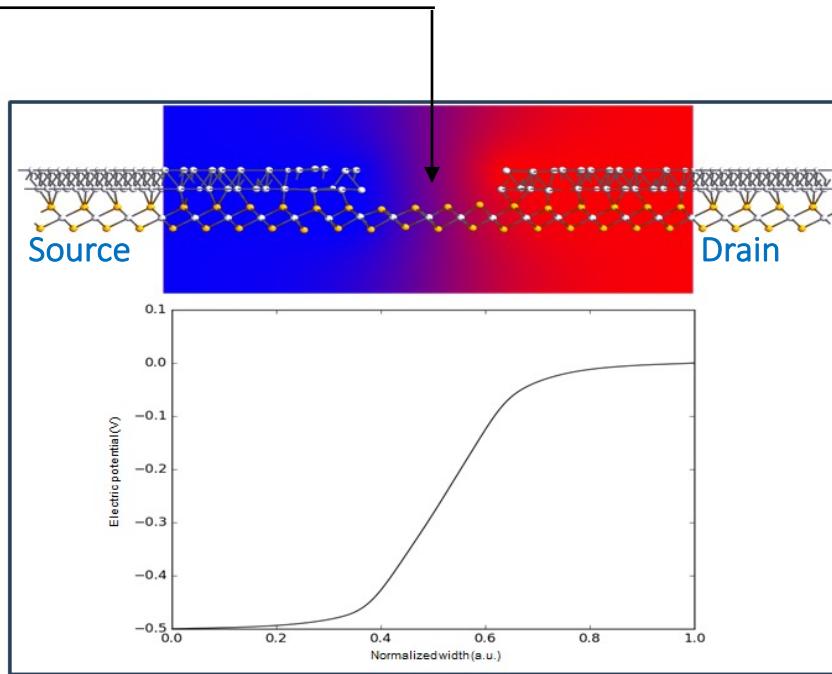
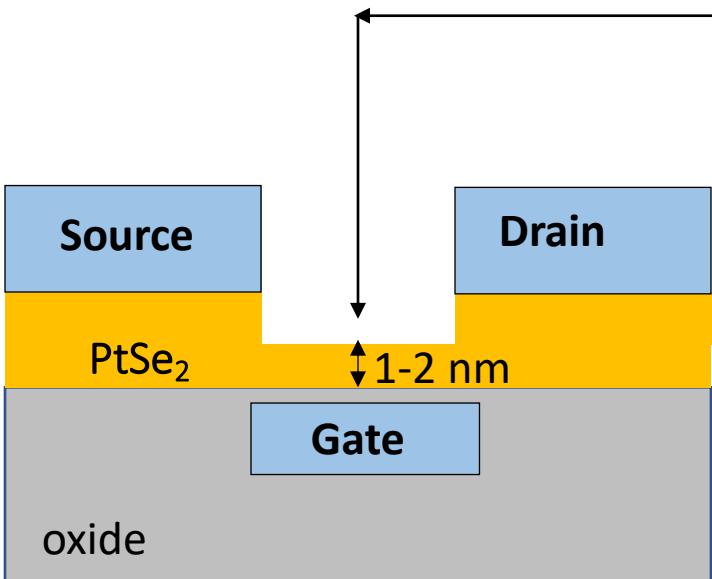


PtSe₂: from monolayer to bulk to metal-contact

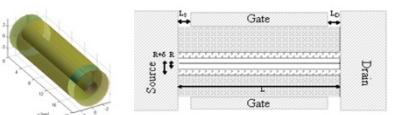


STEP III: inserting the constitutive relations into the full-wave EM simulations

- NOTE: for the modeling of realistic PtSe_2 FET we are trying:
 - A total **ATOMISTIC** simulation (but with a limit of 5k atoms-domain)
 - A full-wave **EM** simulation (extreme scale contrast)
 - An interfaced **ATOMISTIC-EM** model/simulation



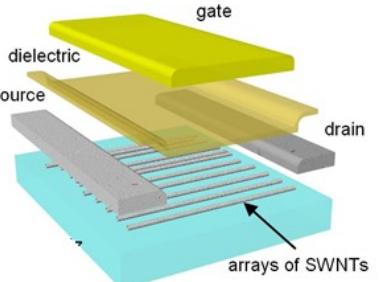
- Results are in the context of a International Project



INPUT DATA

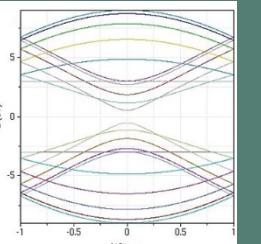
ELECTRICAL DATA

- geometry/shapes
- electrical bias
- external field
- dielectric const.
- metal work function



PRELIMINARY CALCULATION

- CNT electron/hole
- effective mass
- DISP. CURVES
- charge normalization



$$m_{\text{eff.}} = \frac{\hbar^2}{d^2 E} \frac{d^2}{dk^2}$$

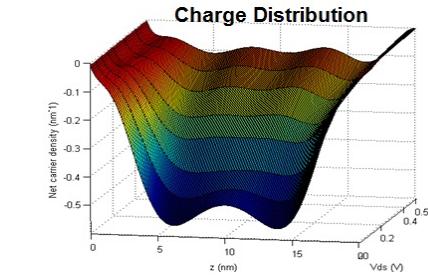
QUANTUM PARAMETERS

- chirality
- radius/length
- Fermi levels
- SW or MW-CNT
- CNT array
- CNT cascade
- energy range
- ...

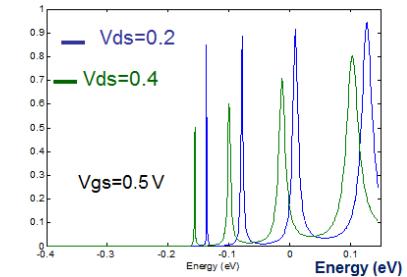
NUMERICAL CONSTRAINTS

- space step
- energy step
- no. iterations
- ...

CNT DESIGN TOOLS I

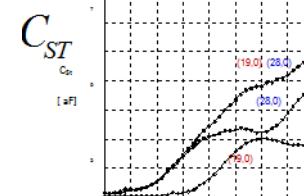


Electronic Transmittivity

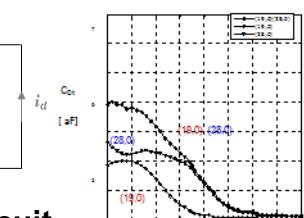


OUTPUT DATA

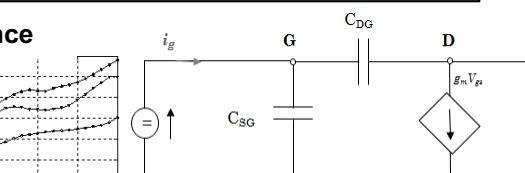
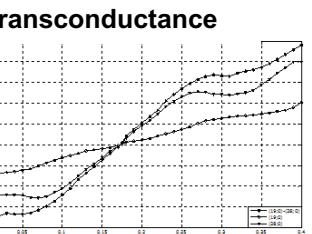
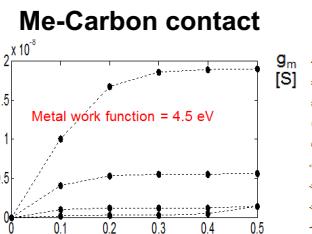
capacitances



C_{DT}

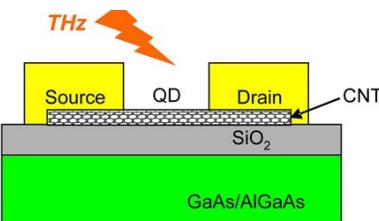
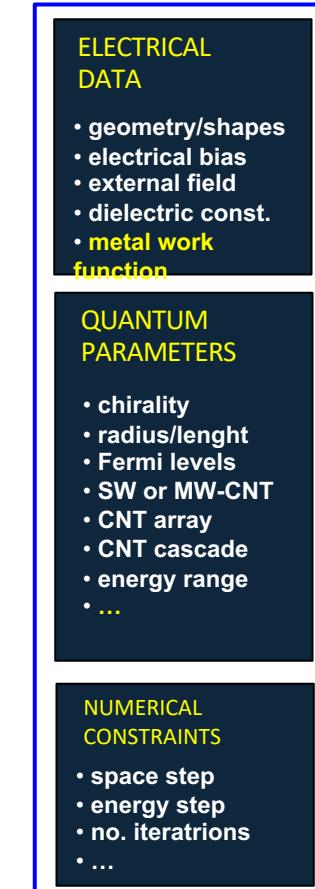


CNT-FET equivalent circuit

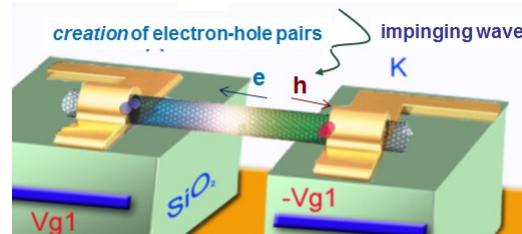
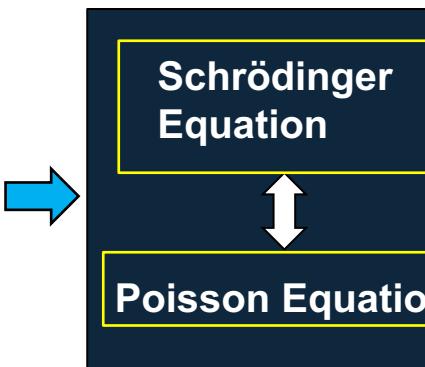


CNT DESIGN TOOLS II

CNT DEVICES

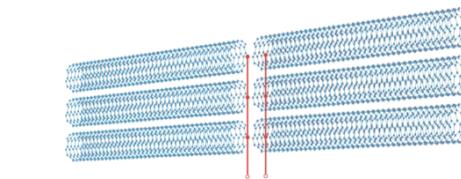
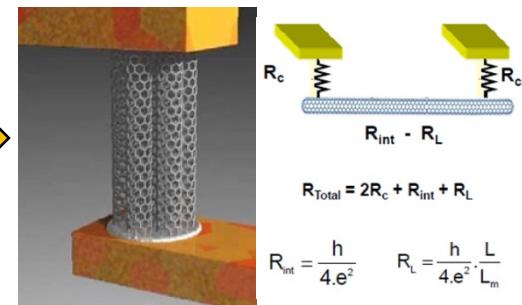


THZ DETECTORs,
PN-JUNCTIONs, QUANTUM
DOTS



PHOTOGENERATION OF CURRENT

CNT INTERCONNECTS



CNT NANOANTENNAS

GRAPHENE DEVICES DESIGN TOOLS III

GRAPHENE/GNR CIRCUITS & FET DESIGN SOFTWARE

ELECTRICAL DATA

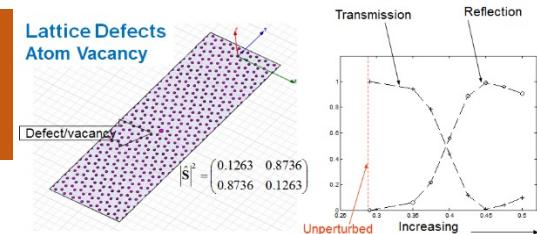
- geometry/shapes
- electrical bias
- external field
- dielectric const.
- metal work function

QUANTUM PARAMETERS

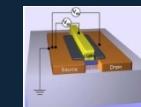
- width/length
- Fermi levels
- metallic/semimetallic/semiconducting GNR

NUMERICAL CONSTRAINTS

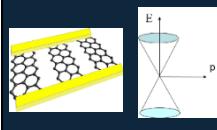
- space step
- energy step
- no. iterations
- adaptive energy refinement



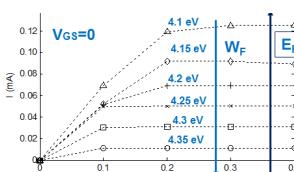
PRELIMINARY CALCULATION



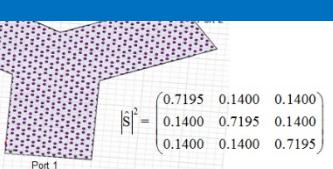
- GNR electron/hole
- effective mass
- DISP. CURVES
- charge normalization



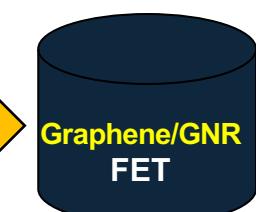
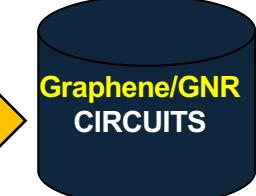
Me-Carbon contact



BALLISTIC freq. domain



S-parameters



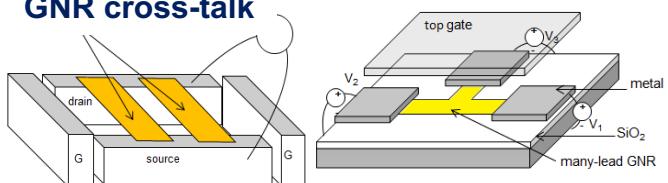
FET Analysis

Dirac Equation

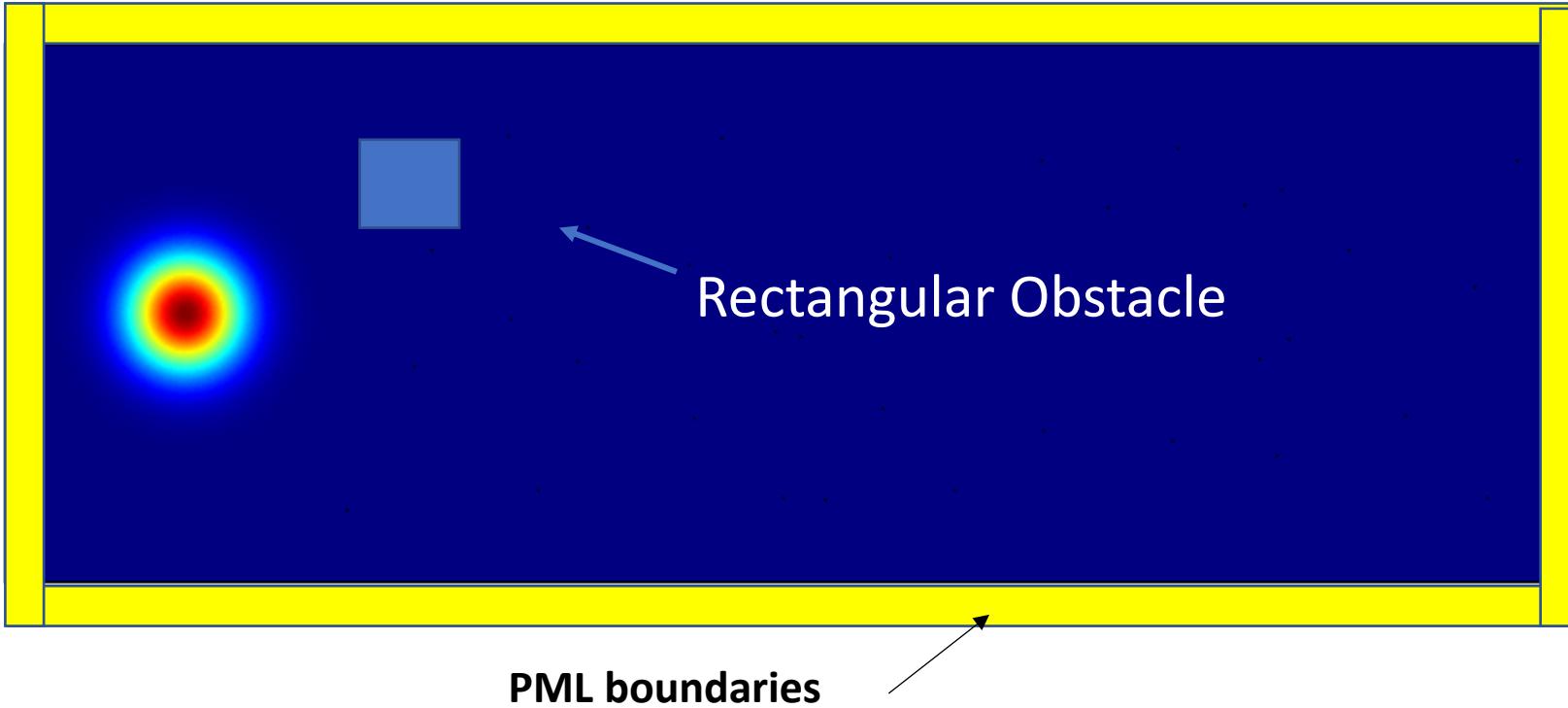
Poisson Equation

- FEM-FD based algorithms
- Iterative process
- SIMULTANEOUS SOLUTION
- optimization
- convergence control
- MATLAB code
- ...

GNR cross-talk



scattering/diffraction by obstacles



$$\Delta L_{\min} = 1 \text{ nm}$$

$$\Delta t = 0.5 \text{ fs}$$

$$T_{\max} = 100 \text{ fs}$$

$$n_s(t_0) = 10^{11} \text{ cm}^{-2}$$

$$E_0 = 0.5 \text{ eV}$$

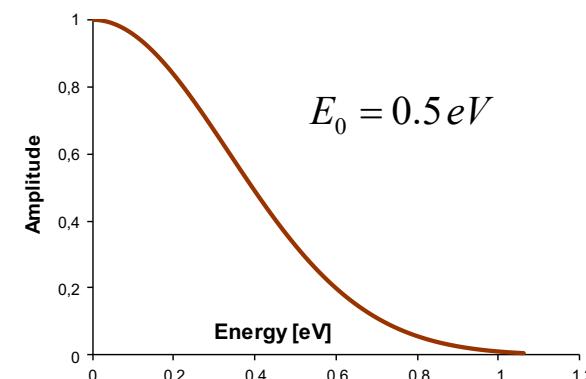
$$\Delta_{PML} = 5 \text{ nm}$$

$$\psi = \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} \quad \begin{cases} \psi_+ = f(x, y; t = 0) \\ \psi_- = 0 \end{cases}$$

$$f(x, y; t = 0) = \psi_N \left\{ \exp\left(\frac{-(x - x_0)^2}{2\sigma_x^2}\right) \cdot \exp\left(\frac{-(y - y_0)^2}{2\sigma_z^2}\right) \right\}$$

$$\int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = 1 \Rightarrow q \int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = Q_0 \Rightarrow \psi_N$$

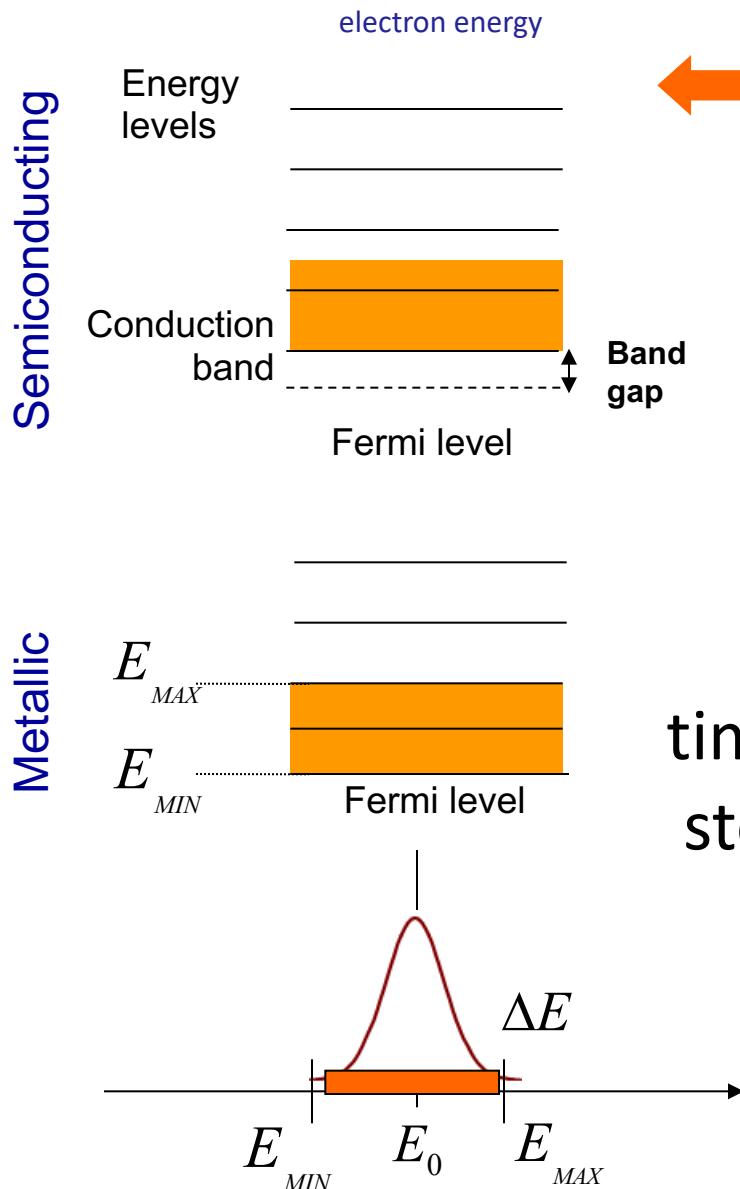
$$n_s(t_0) \Leftrightarrow Q_0$$



Physical Properties



Computational Parameters



$$E_{elec} = \omega\hbar = hf_{elec} \Rightarrow f_{elec} = \frac{E_{elec}}{\hbar} \quad \hbar = \frac{h}{2\pi}$$

$$E_{MAX} \Rightarrow f_{MAX} \quad E_{MIN} \Rightarrow f_{MIN}$$

$$\lambda_{MAX} = \frac{v_F}{f_{MIN}} \quad \lambda_{MIN} = \frac{v_F}{f_{MAX}}$$

$$\Delta L_{grid} \leq \frac{1}{30} \lambda_{MIN}$$

space-step



$$\Delta t_{EM} \leq \frac{\Delta L_{grid}}{2c} \quad c = 3 \cdot 10^8 \text{ m/s}$$

EM grid
(TLM)

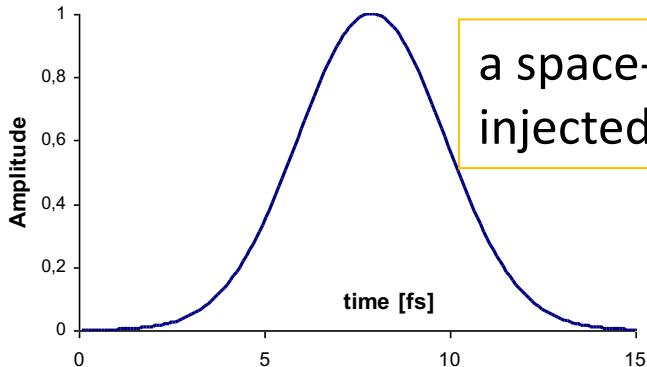
$$\Delta t_{GNR} \leq \frac{\Delta L_{grid}}{v_F} \quad v_F \approx c/300 \text{ m/s}$$

GNR grid

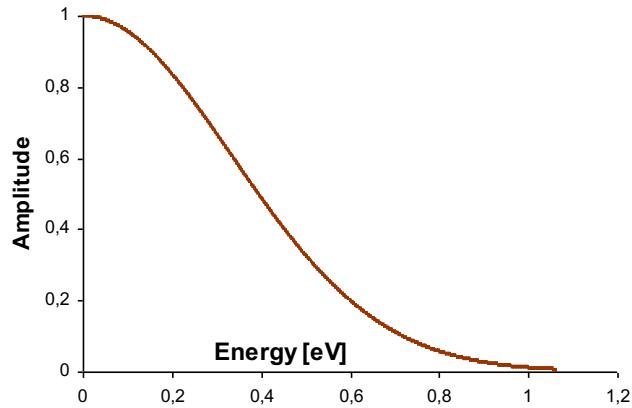
Propagation of a Gaussian pulse for wide-band electron energy

$$\underline{\psi} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} \quad \begin{cases} \psi_1 = f(x, z; t) \\ \psi_2 = -if(x, z; t) \\ \psi_3 = \psi_4 = 0 \end{cases} \quad \int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = 1 \Rightarrow q \int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = Q_T$$

$$f(x, z; t) = A_N \exp\left(\frac{-(t-t_0)^2}{2\sigma_T^2}\right) \left\{ \exp\left(\frac{-(x-x_0)^2}{2\sigma_x^2}\right) \cdot \exp\left(\frac{-(z-z_0)^2}{2\sigma_z^2}\right) \right\}$$



a space-time pulse is injected



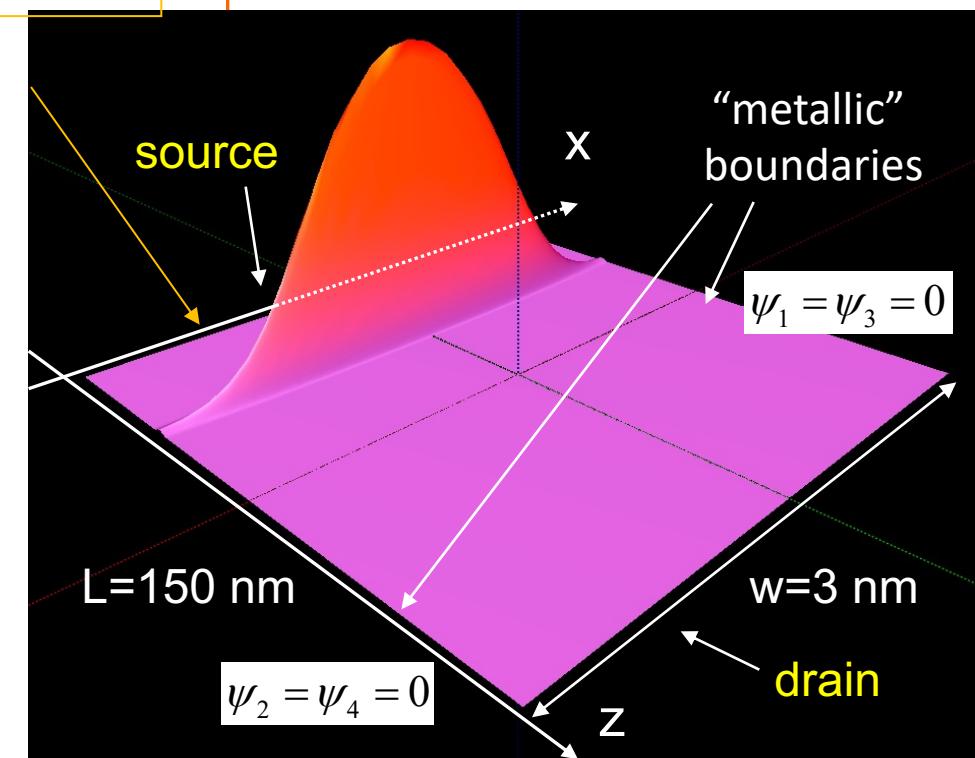
Computational Parameters

$$E_{MAX} = 1 \text{ eV} \quad f_{MAX} = 289 \text{ THz}$$

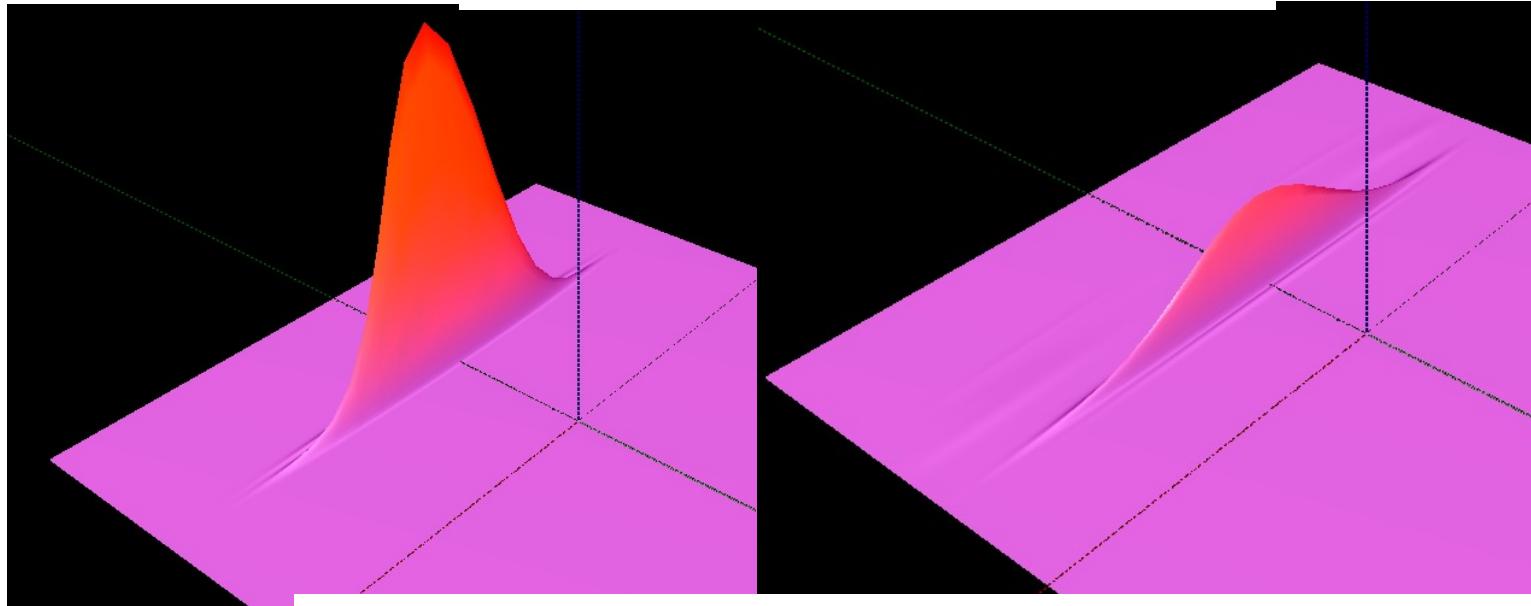
$$\sigma_T = 2 / f_{MAX}$$

$$\lambda_{MIN} = 5 \text{ nm} \Rightarrow \Delta L_{grid} = \frac{1}{32} \lambda_{MIN} = 0.12 \text{ nm}$$

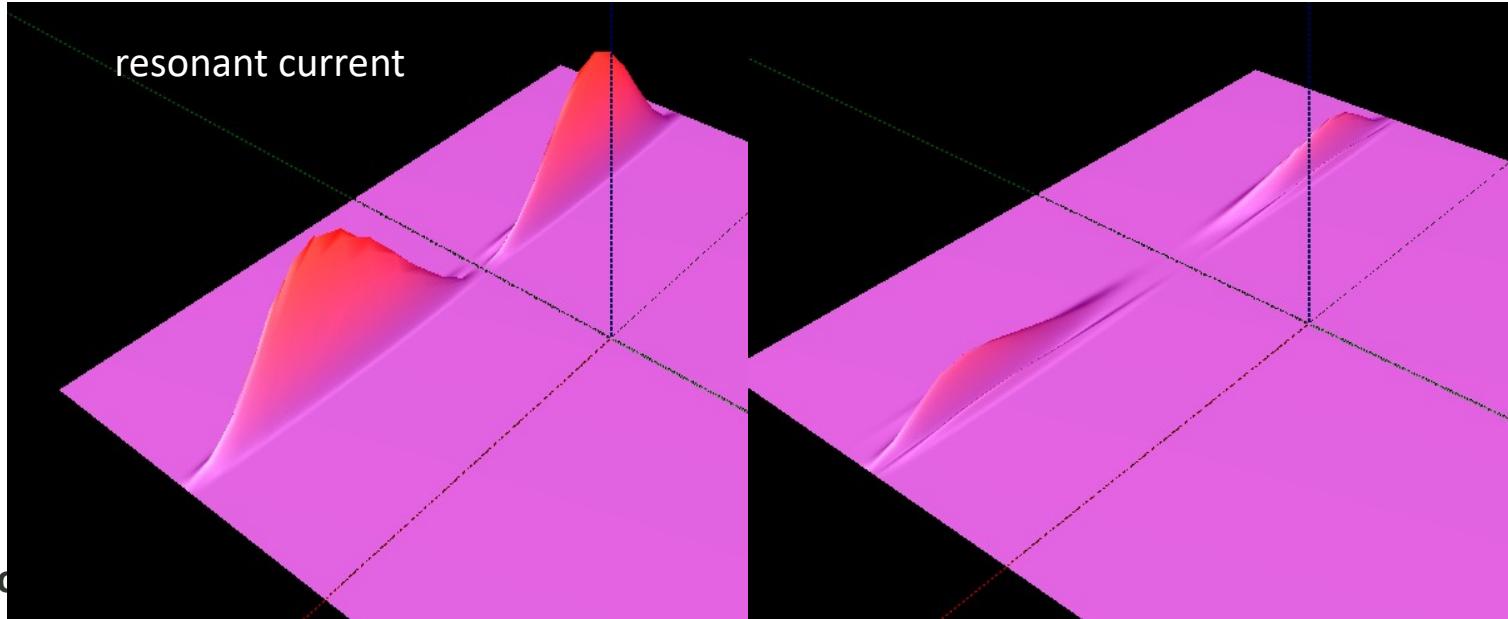
$$\Delta t_{EM} = \frac{\Delta L_{grid}}{2c_0} = 2 * 10^{-4} \text{ fs} \quad \Delta t_{GNr} = \frac{\Delta L_{grid}}{v_F} = 0.12 \text{ fs}$$



Propagation of $|J_z|$

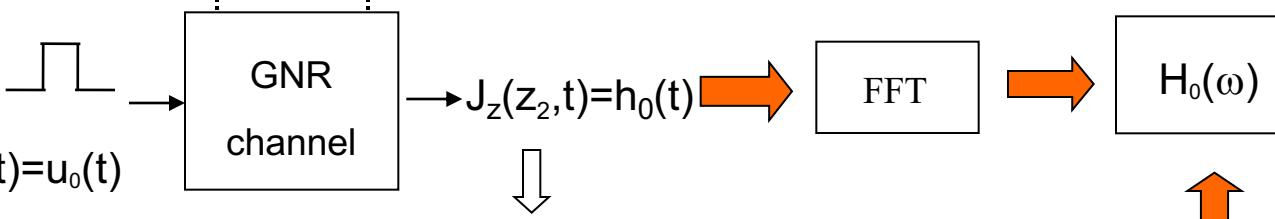
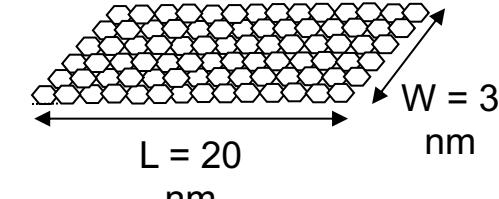
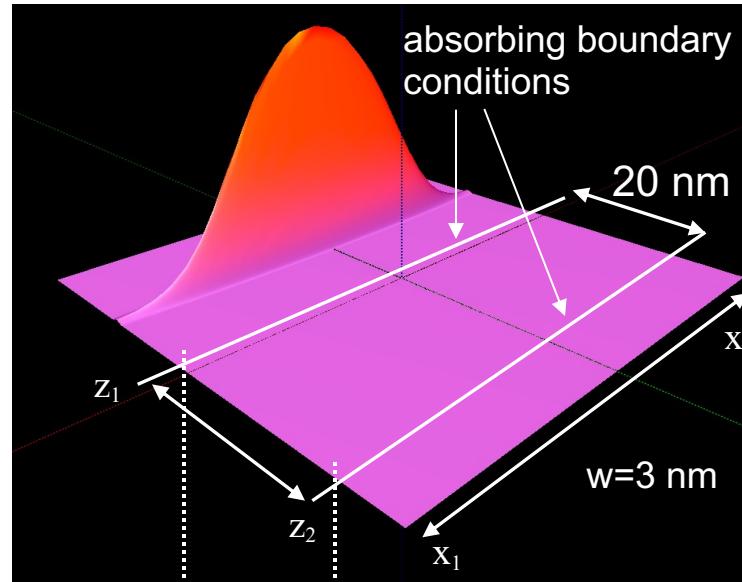
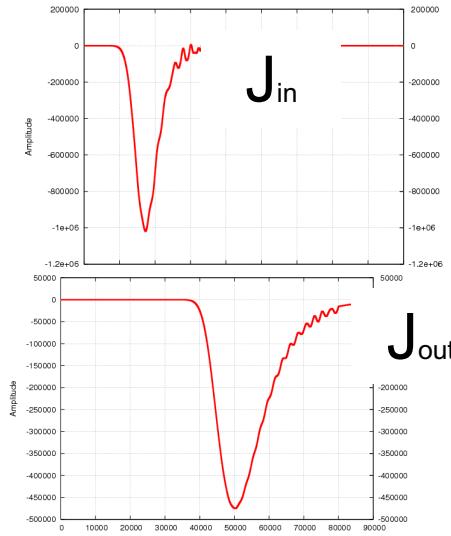


Propagation of $|J_x|$



Transmittivity of a Graphene Channel

$$J_z(z; t) = \int_{x_1}^{x_2} J_z(x, z; t) dx [A]$$

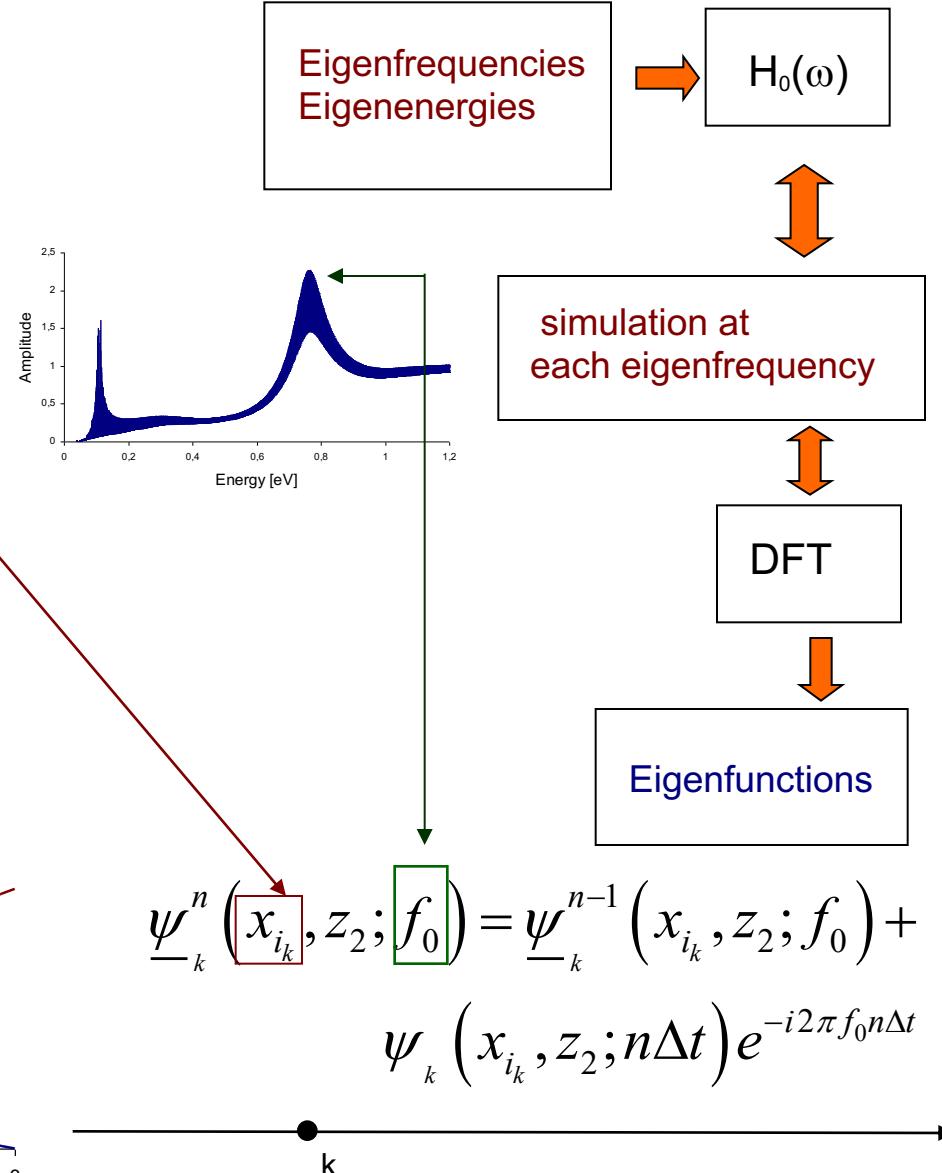
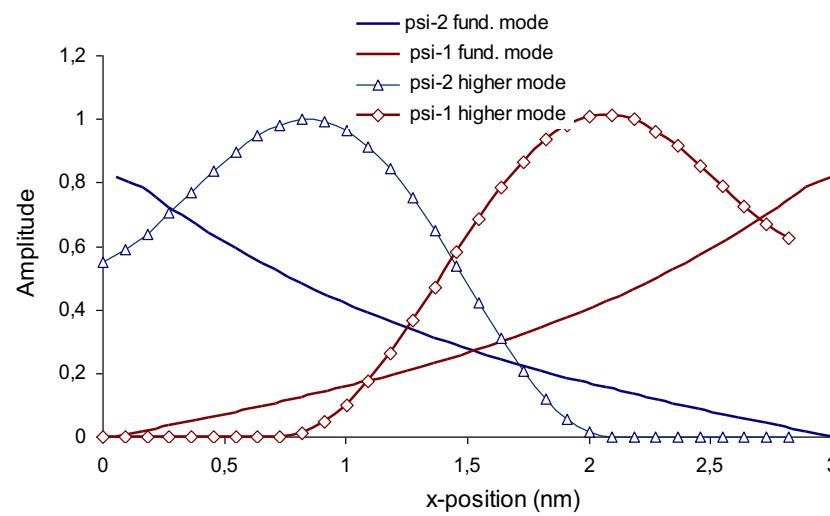
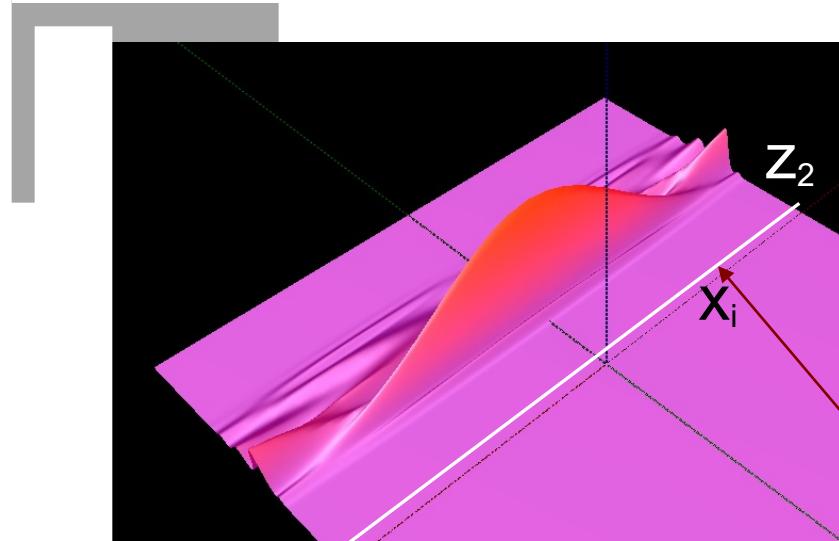


the amplitude peaks
of $H(\omega)$ are eigenenergies

$$H_0(\omega) = \frac{J_z(z_2, \omega)}{J_z(z_1, \omega)} = \frac{F\{J_z(z_2, t)\}}{F\{J_z(z_1, t)\}}$$

Transmittivity

mode profile



a DFT at each point in the problem space at each of the eigenenergies f_0 is carried out during the simulation

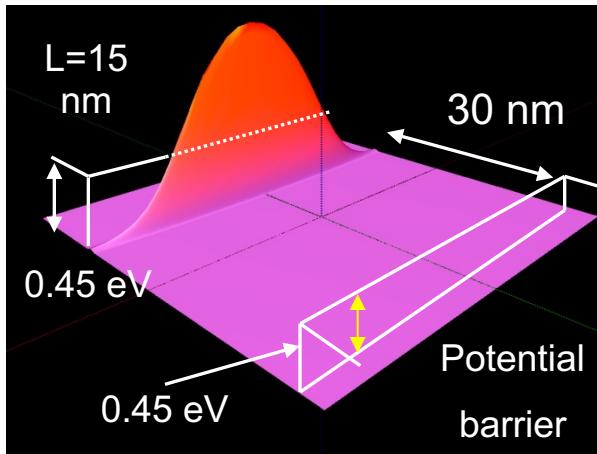
self-generated EM field in the presence of a potential barrier

$$E_{MAX} = 1 \text{ eV}$$

$$\Delta t_{EM} \leq \frac{\Delta L_{grid}}{2c} \quad c = 3 \cdot 10^8 \text{ m / s}$$

$$\Delta t_{GNR} \leq \frac{\Delta L_{grid}}{v_F} \quad v_F \approx c / 300 \text{ m / s}$$

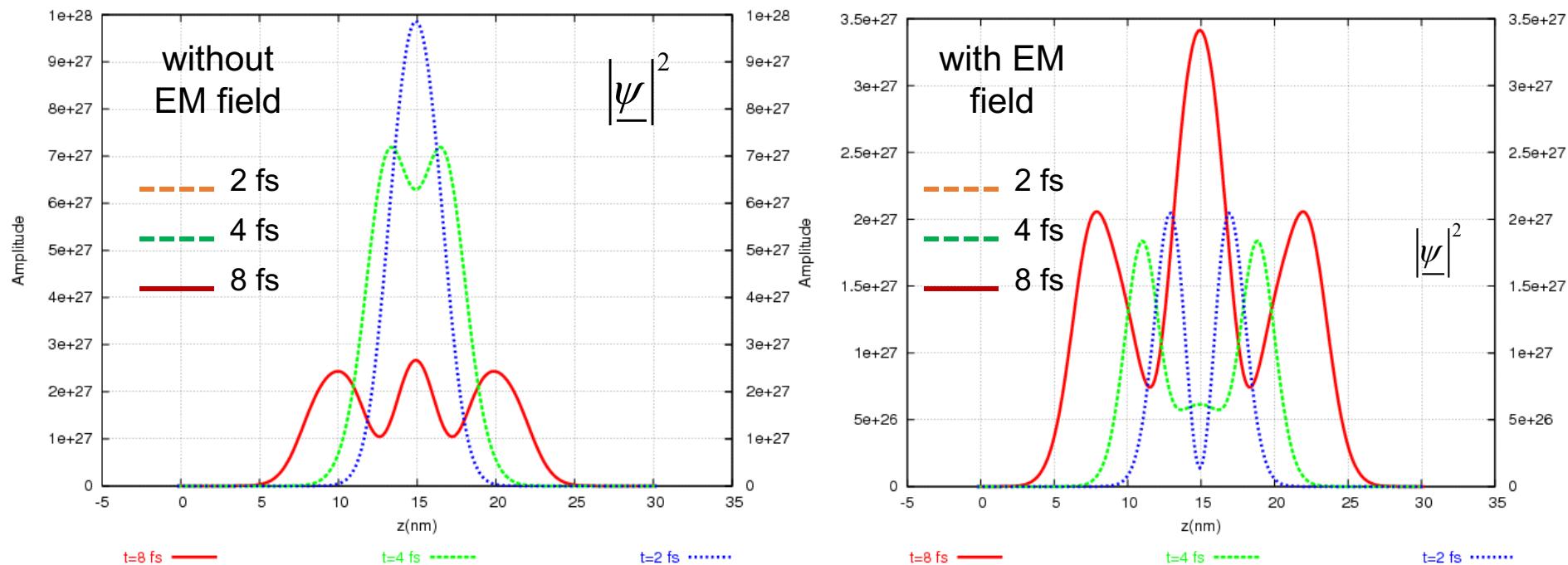
$$\Delta t_{EM} \leq \frac{\Delta t_{GNR}}{600}$$



$$\mathbf{k} = \mathbf{p} - q\mathbf{A}$$



change in the
dynamics



CNT wavepacket dynamics with or without the self-generated EM field

